# 4. THE MAGNETIC VECTOR POTENTIAL A

In electrostatics we are familiar with V, the scalar potential – it is a very useful quantity with which to solve problems as it is easier to handle than **E**; and we recall $\mathbf{E} = -\nabla V$.

In magnetostatics we can write

$$\mathbf{B} = \nabla \mathbf{x} \mathbf{A}$$

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| The magnetic field **B** is the curl of a magnetic vector potential **A**. |

Nature requires that $\nabla .\mathbf{B} = 0$. This is automatically assured by the vector algebra since divergence of curl is always zero (vector identity (9) inside front cover of Griffiths.)

## 4.1 Ampere's law for A

In section 3.1 we derived Ampere's law:

$$\nabla \mathbf{x} \mathbf{B} = \mu_0 \mathbf{J} \qquad \text{Ampère's law}$$

Now if $\mathbf{B} = \nabla \mathbf{x} \mathbf{A}$,

$$\nabla \mathbf{x} (\nabla \mathbf{x} \mathbf{A}) = \mu_0 \mathbf{J}$$

and we can use a standard vector identity (#11) to re-write this

$$\nabla\mathbf{x}(\nabla\mathbf{x}\mathbf{A}) = \nabla(\nabla.\mathbf{A}) - \nabla^2\mathbf{A}$$

This term $\nabla.\mathbf{A}$ can always be made equal to zero since we can always add a term $\nabla\lambda$ to $\mathbf{A}$ ensure $\nabla.\mathbf{A} \equiv 0$, that is, we can write $\mathbf{A} = \mathbf{A_0} + \nabla\lambda$

With $\nabla.\mathbf{A} = 0$, Ampère's law is

$$\nabla^2\mathbf{A} = -\mu_0\mathbf{J} \qquad \text{Ampere's law}$$

which has the same form as Poisson's equation in electrostatics

$$\nabla^2 V = -\frac{\rho}{\varepsilon_0} \qquad \text{Poisson's Equation}$$

There are clear similarities – at least in the form of the mathematics.

But, there are major differences too:

• The sources of magnetic fields (currents!) are directional, i.e. vector quantities, so the potential can’t be a scalar.

• In electrostatics, V is derived from (and understood as) the **work done** per unit charge in moving a test charge in electric field **E**. But magnetic forces do no work. The ‘intuitive’ relationship we can appreciate in the case of **E** and V is therefore not available.

## 4.2 Reality of A: The Aharonov-Bohm Effect

In 1957, Aharonov and Bohm predicted that if a quantum-mechanical wave-like particle were split into two partial waves, which then passed on either side of a region of magnetic field (eg a solenoid), then there would be a phase shift between the two waves. Thus, if they later recombined, the interference between them would depend on the amount of flux enclosed between the two waves.

In fact, it is not the magnetic *field* **B** that actually matters, but the magnetic vector potential **A**. The wave/particle can even travel in a region where **B** is zero, provided that within the path there is some region containing magnetic flux!

This seems strange, as **A** is not a well-defined quantity - there are many choices of **A** for a given **B**, depending on the "gauge" you choose. However, when the waves recombine, all that matters is the loop integral of **A.dl**, which is the same as the flux through the ring (by Stoke's Theorem).

The net result is that the phase accumulated is 2π for each *h*/*e* of flux enclosed within the two paths. This is the Aharonov-Bohm Effect.

The same applies if a wave travels around a loop (being reflected or scattered at various points).