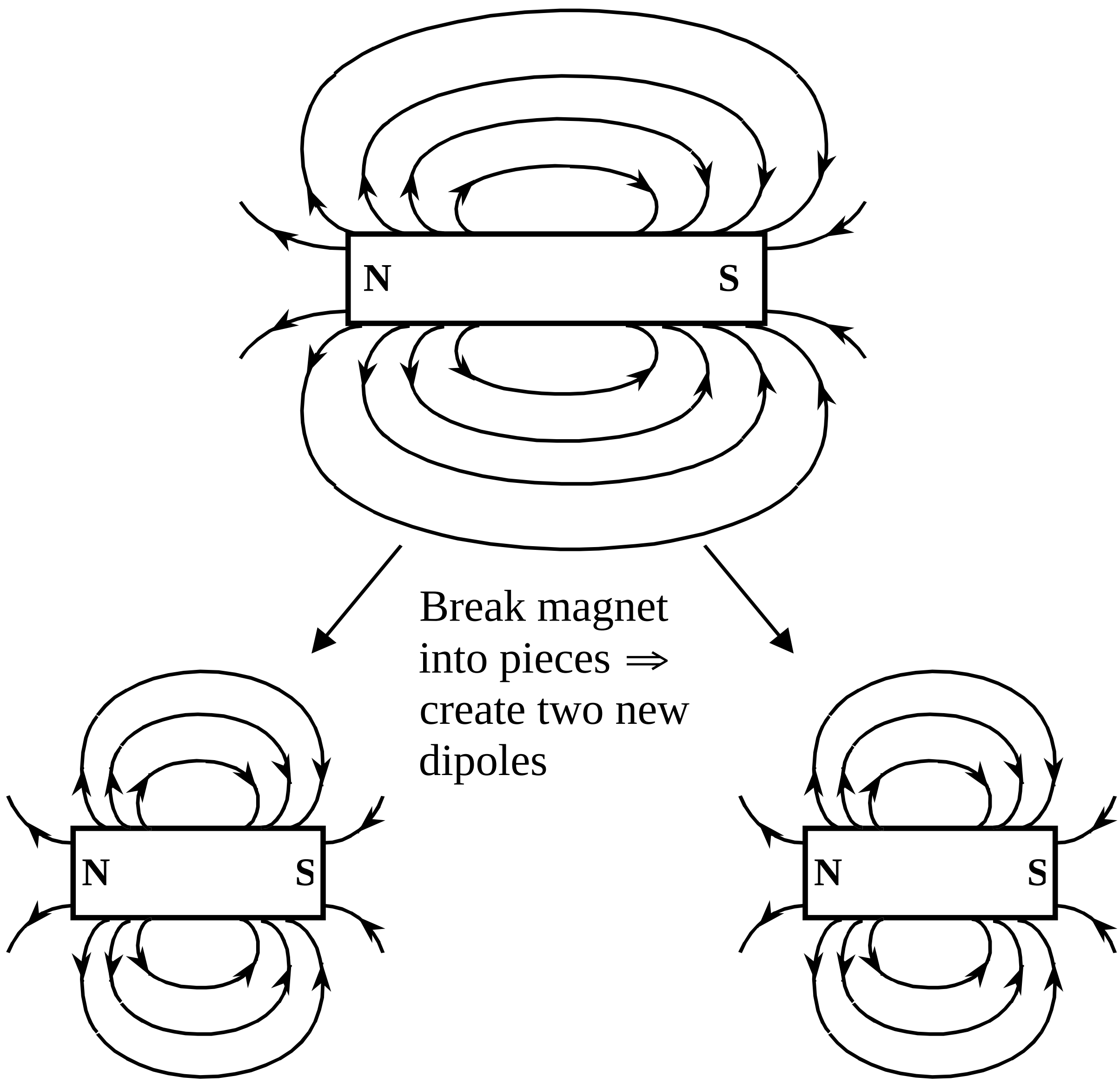
5. MAGNETIC DIPOLES, FORCES AND TORQUES

5.1 What is a magnetic dipole?

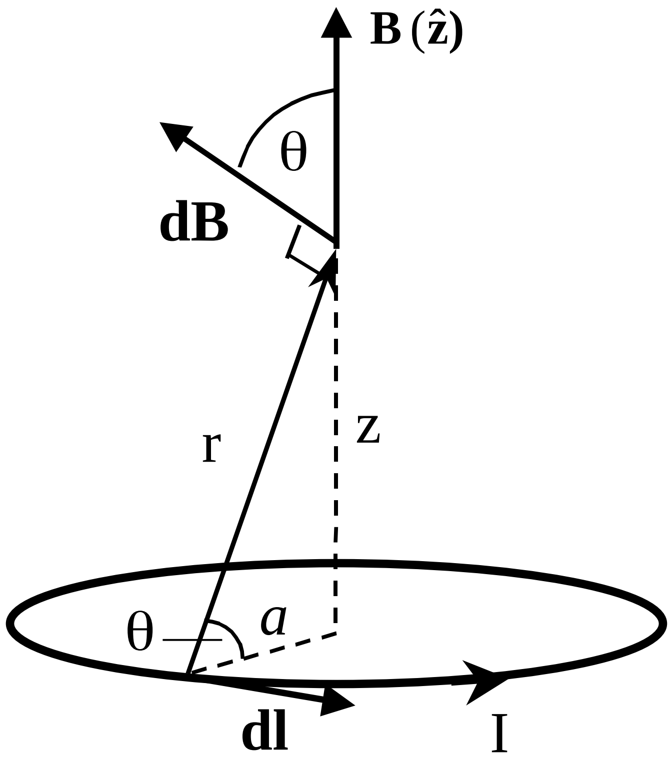
In the previous section we said that magnetic monopoles are not found in nature. (Actually, monopoles are allowed by theory but theory also says that a monopole colliding with a proton cause the proton to decay in which case there are not very many in this part of the Universe at least…)



However many successively smaller pieces the magnet is divided into there is always a N-S pole pair – a **dipole**.

5.2 Dipole field from a current loop

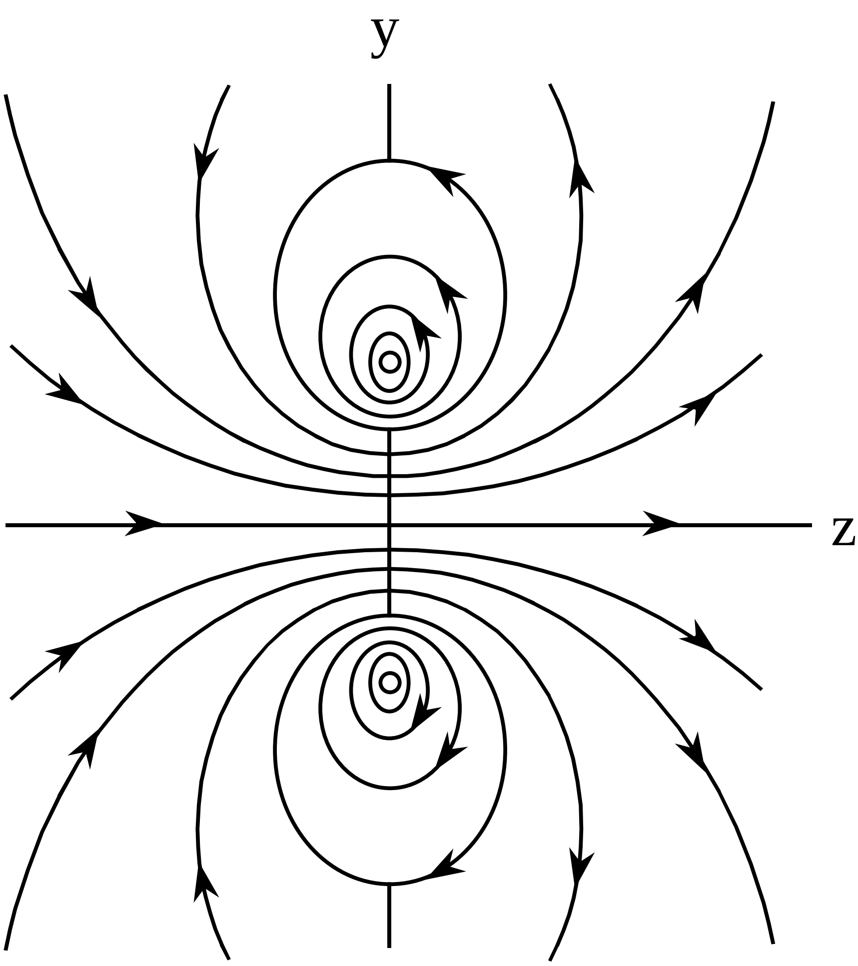
In section 2.2.2 we used the Biot-Savart law to calculate B on the axis of a current carrying loop of radius *a*:



and found

$$\mathbf{B} = \frac{\mu_0 I a^2}{2(a^2 + z^2)^{3/2}}\ \hat{\mathbf{z}}$$

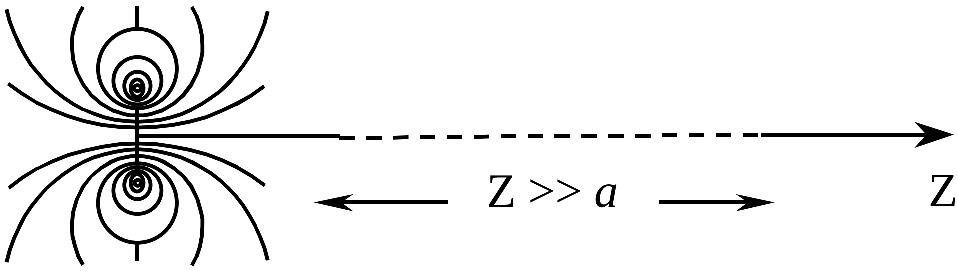
If we view the field due to this loop (this view, is a a side-on section through the loop across a diameter; the plane of the loop is perpendicular to the page):



Current direction: top of loop: I is out of page

bottom of loop: I is into page

If we look at the field produced by the loop *at distances large* compared to the size of the loop, z >> *a* where *a* is the loop radius:



The field looks the same as the (electric) field of the electric dipole.

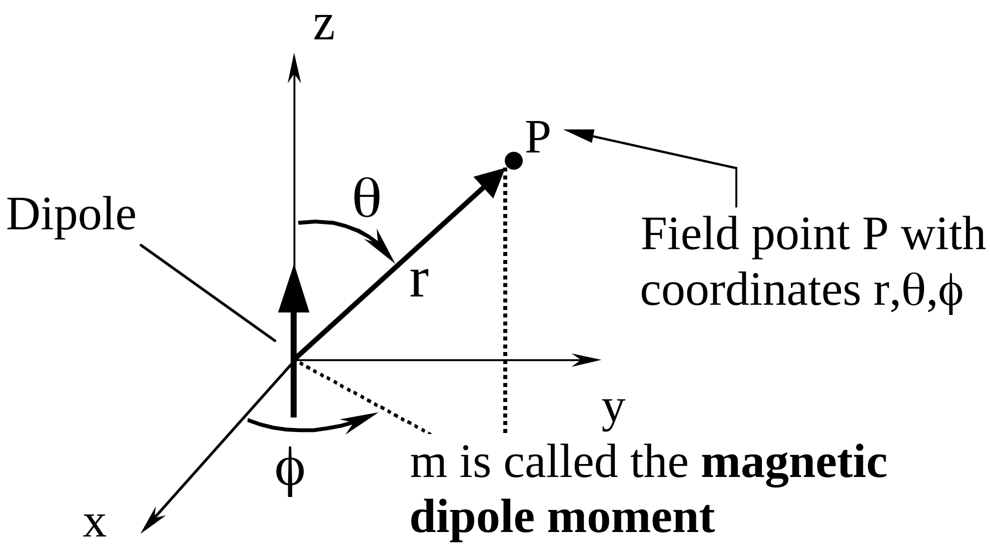
Compare the expressions for magnetic and electric dipole fields:

$$\mathbf{E} = \frac{p}{4\pi\varepsilon_0 r^3}(2\cos\theta\,\hat{\mathbf{r}} + \sin\theta\,\hat{\theta})$$

Electric dipole

$$\mathbf{B} = \frac{\mu_0 m}{4\pi r^3}(2\cos\theta\,\hat{\mathbf{r}} + \sin\theta\,\hat{\theta})$$

Magnetic dipole



$$\mathbf{B} = \frac{\mu_0 m}{4\pi r^3}(2\cos\theta\,\hat{\mathbf{r}} + \sin\theta\,\hat{\theta})$$

5.3 Compare electric and magnetic dipoles

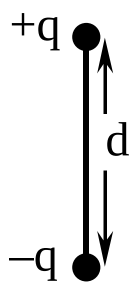
To understand what **m** is compare it to the electric dipole moment:

Electric dipole:

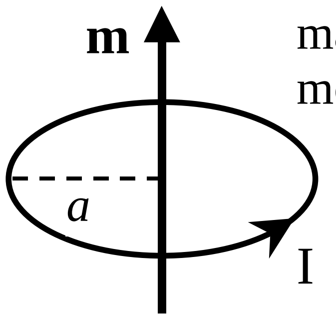
$$\mathbf{p} = q\mathbf{d}$$

electric dipole moment

vector pointing from –ve to +ve charge



Magnetic dipole:

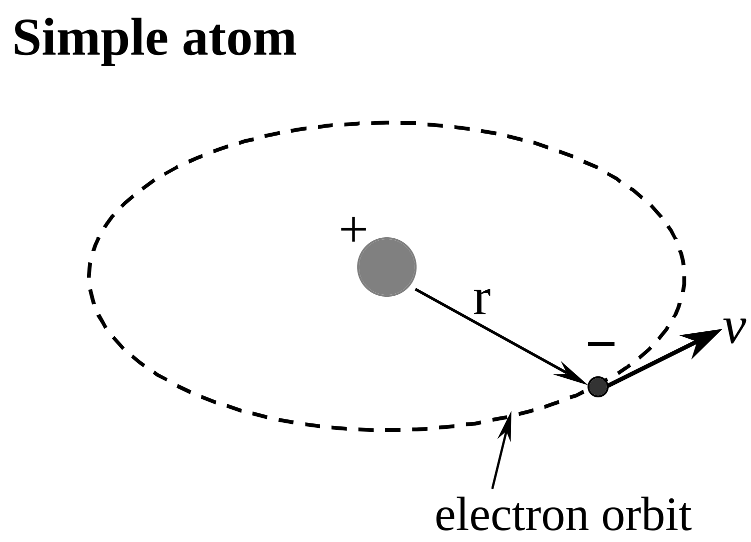


magnetic dipole moment

$\mathbf{m} = I\pi a^2$ direction of **m** is given by right hand rule

5.4 Atomic dipole moment

If you have been wondering why we bothered to consider the current loop and magnetic dipole here's why…think of a very small current loop:



There is a current in the 'atomic loop' in direction opposite to *v* (see Tipler p906).

$$I = \frac{ev}{2\pi r}$$

where e is the electron charge. The dipole moment of a current loop is $m = IA$ and

$$m = IA = \frac{ev}{2\pi r}\pi r^2 = \frac{evr}{2}$$

The orbital angular momentum **L** is **L = rxp** with magnitude

$$L = rp\sin 90^0 = rp = mvr$$

electron mass

So that

$$\mathbf{m} = \frac{-e}{2m}\mathbf{L}$$

electron magnetic moment

electron mass

By convention, we use $\boldsymbol{\mu}$ rather than **m** for the electron dipole moment:

$$\boxed{\boldsymbol{\mu} = \frac{-e}{2m}\mathbf{L}}$$

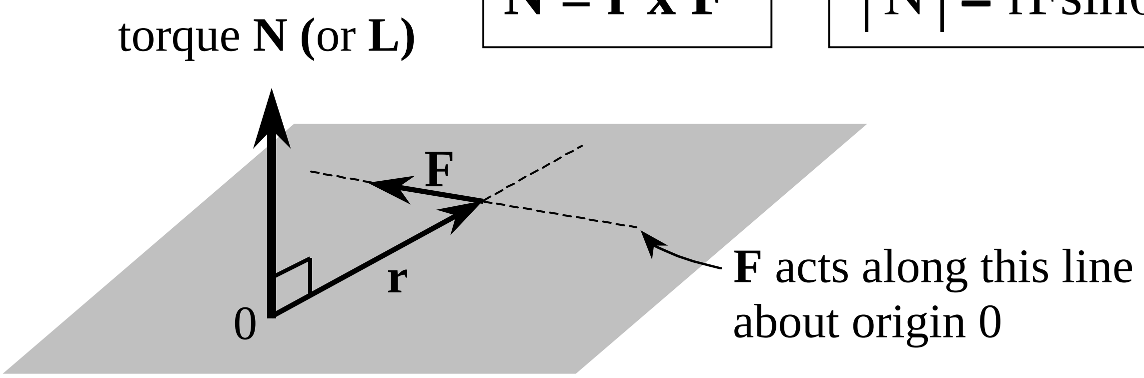
orbital magnetic moment of electron

5.5 Torque on a current loop

A torque is a twisting force or **moment** about an axis.

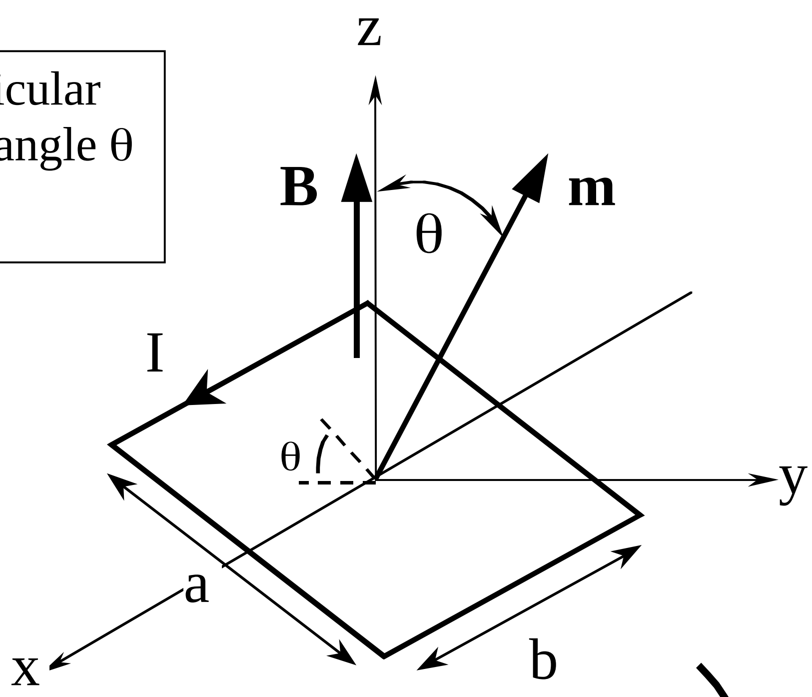
Reminder about torque:

$$\boxed{\mathbf{N} = \mathbf{r} \times \mathbf{F}} \qquad \boxed{|N| = rF\sin\theta}$$



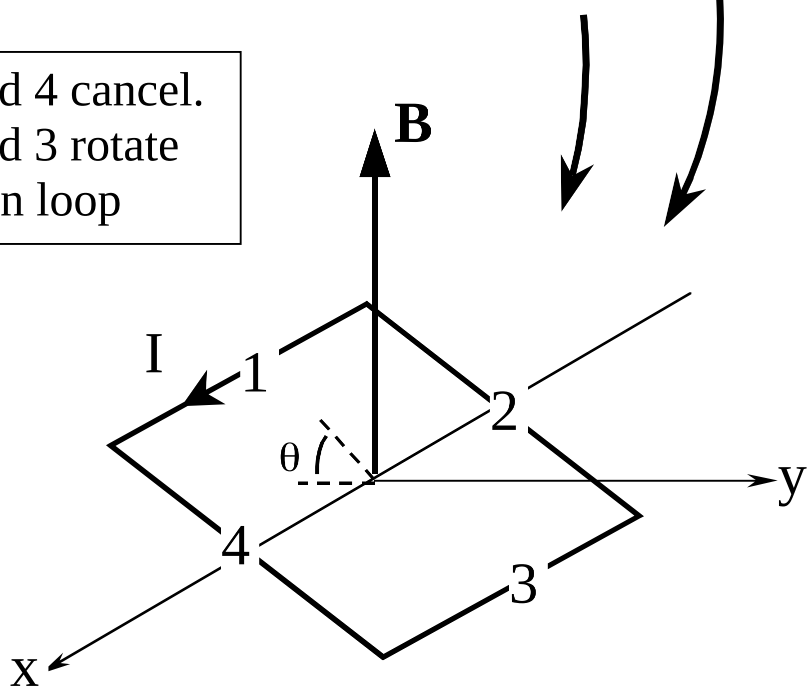
Take a rectangular current-carrying loop (it's easier than the circular loop we are interested in) and find the torque on it in a magnetic field B in the z-direction:

moment **m** is perpendicular to current loop and at angle θ to magnetic field **B**

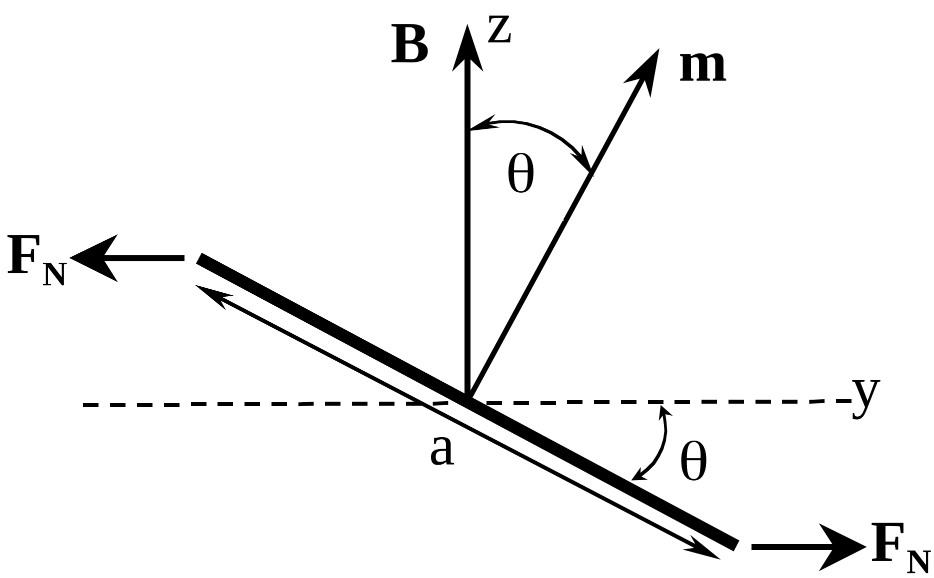


We can see the force on each side of loop from d**F** = Id**l**x**B**

Forces on sides 2 and 4 cancel.
Forces on sides 1 and 3 rotate the loop ⇒ torque on loop



Side on (looking *along* the sides of length b) the loop is:



Notice the forces tend to pull the plane of loop cross-wise to the field; the magnetic dipole moment **m** is $\therefore$ aligned parallel to **B**

The torque is

$$N = aF\sin\theta \quad \text{along the x-direction,}$$

i.e.

$$\mathbf{N} = aF\sin\theta\ \hat{\mathbf{x}}$$

and since

$$d\mathbf{F} = Id\mathbf{l}\mathbf{x}\mathbf{B}$$

$$F = IbB$$

(the sides b of the loop give the torque) we have

$$\mathbf{N} = aF\sin\theta \ \hat{\mathbf{x}}$$

$$\mathbf{N} = mB\sin\theta \ \hat{\mathbf{x}}$$

dipole moment $m = I \times (\text{loop area}) = Iab$

So

$$\mathbf{N} = \mathbf{m} \times \mathbf{B}$$

torque = magnetic dipole moment x field strength

(Again note the comparison to the electrical case: **N = p x E** for electric dipole moment **p** in electric field **E.** The electric dipole lines up parallel to the direction of **E** due the action of the torque **N**.)