5. MAGNETIC DIPOLES, FORCES AND TORQUES

5.1 What is a magnetic dipole?

In the previous section we said that magnetic monopoles are not found in nature. (Actually, monopoles are allowed by theory but theory also says that a monopole colliding with a proton cause the proton to decay \( \rightarrow \) in which case there are not very many in this part of the Universe at least…)

[Diagram of a magnetic dipole and its breakdown into two new dipoles]

Break magnet into pieces ⇒ create two new dipoles
However many successively smaller pieces the magnet is divided into there is always a N-S pole pair – a **dipole**.

**5.2 Dipole field from a current loop**

In section 2.2.2 we used the Biot-Savart law to calculate $B$ on the axis of a current carrying loop of radius $a$:

$$B = \frac{\mu_0 I a^2}{2(a^2 + z^2)^{3/2}} \hat{z}$$
If we view the field due to this loop (this view, is a side-on section through the loop across a diameter; the plane of the loop is perpendicular to the page):

Current direction: top of loop: I is out of page  
bottom of loop: I is into page

If we look at the field produced by the loop at distances large compared to the size of the loop, $z \gg a$ where $a$ is the loop radius:
The field looks the same as the (electric) field of the electric dipole. Compare the expressions for magnetic and electric dipole fields:

\[
E = \frac{p}{4\pi\varepsilon_0 r^3} (2\cos \theta \hat{r} + \sin \theta \hat{\theta}) \\
B = \frac{\mu_0 m}{4\pi r^3} (2\cos \theta \hat{r} + \sin \theta \hat{\theta})
\]

Electric dipole  
Magnetic dipole
5.3 Compare electric and magnetic dipoles

To understand what $\mathbf{m}$ is compare it to the electric dipole moment:

**Electric dipole:**

\[ \mathbf{p} = qd \]

Electric dipole moment vector pointing from –ve to +ve charge

**Magnetic dipole:**

\[ \mathbf{m} = I\pi a^2 \]

direction of $\mathbf{m}$ is given by right hand rule
5.4 Atomic dipole moment

If you have been wondering why we bothered to consider the current loop and magnetic dipole here’s why…think of a very small current loop:

There is a current in the ‘atomic loop’ in direction opposite to \( \nu \) (see Tipler p906).

\[
I = \frac{ev}{2\pi r}
\]

where \( e \) is the electron charge. The dipole moment of a current loop is \( m = IA \) and

\[
m = IA = \frac{ev}{2\pi r} \pi r^2 = \frac{e\nu r}{2}
\]
The orbital angular momentum $L$ is $L = r \mathbf{p}$ with magnitude

$$L = rp \sin 90^\circ = rp = mvr$$

So that

$$\mathbf{m} = \frac{-e}{2m} L$$

By convention, we use $\mathbf{\mu}$ rather than $\mathbf{m}$ for the electron dipole moment:

$$\mathbf{\mu} = \frac{-e}{2m} L$$

5.5 Torque on a current loop

A torque is a twisting force or \textbf{moment} about an axis.

Reminder about torque:

$$\mathbf{N} = r \times \mathbf{F}$$

$$|\mathbf{N}| = rF\sin \theta$$

F acts along this line about origin 0
Take a rectangular current-carrying loop (it’s easier than the circular loop we are interested in) and find the torque on it in a magnetic field $\mathbf{B}$ in the $z$-direction:

\[ \text{moment } \mathbf{m} \text{ is perpendicular to current loop and at angle } \theta \text{ to magnetic field } \mathbf{B} \]

\[
\begin{align*}
\text{Forces on sides 2 and 4 cancel.} \\
\text{Forces on sides 1 and 3 rotate the loop } \Rightarrow \text{ torque on loop}
\end{align*}
\]

We can see the force on each side of loop from $d\mathbf{F} = I dl \times \mathbf{B}$
Side on (looking *along* the sides of length b) the loop is:

\[ \mathbf{F}_N = \mathbf{a} \cdot \mathbf{F} \sin \theta \]
\[ \mathbf{N} = \mathbf{a} \mathbf{F} \sin \theta \]

*Notice* the forces tend to pull the plane of loop cross-wise to the field; the magnetic dipole moment \( \mathbf{m} \) is \( \therefore \) aligned parallel to \( \mathbf{B} \)

The torque is

\[ \mathbf{N} = \mathbf{a} \mathbf{F} \sin \theta \text{ along the x-direction,} \]

i.e.

\[ \mathbf{N} = \mathbf{a} \mathbf{F} \sin \theta \hat{x} \]

and since

\[ d\mathbf{F} = I d\mathbf{l} \times \mathbf{B} \]
\[ \mathbf{F} = I b \mathbf{B} \]

(the sides b of the loop give the torque)
we have

\[ \mathbf{N} = aF \sin \theta \ \hat{x} \]
\[ \mathbf{N} = mB \sin \theta \ \hat{x} \]

dipole moment \( m = I \times \text{(loop area)} = \text{Iab} \)

So

\[ \mathbf{N} = \mathbf{m} \times \mathbf{B} \]

torque = magnetic dipole moment \( \times \) field strength

Again note the comparison to the electrical case: \( \mathbf{N} = \mathbf{p} \times \mathbf{E} \)
for electric dipole moment \( \mathbf{p} \) in electric field \( \mathbf{E} \). The electric dipole lines up parallel to the direction of \( \mathbf{E} \) due the action of the torque \( \mathbf{N} \).