

6. MAGNETIC MATERIALS

6.1 Atomic sized current loops

We've emphasised throughout this course the central point that *magnetic fields are produced by electric charges in motion*.

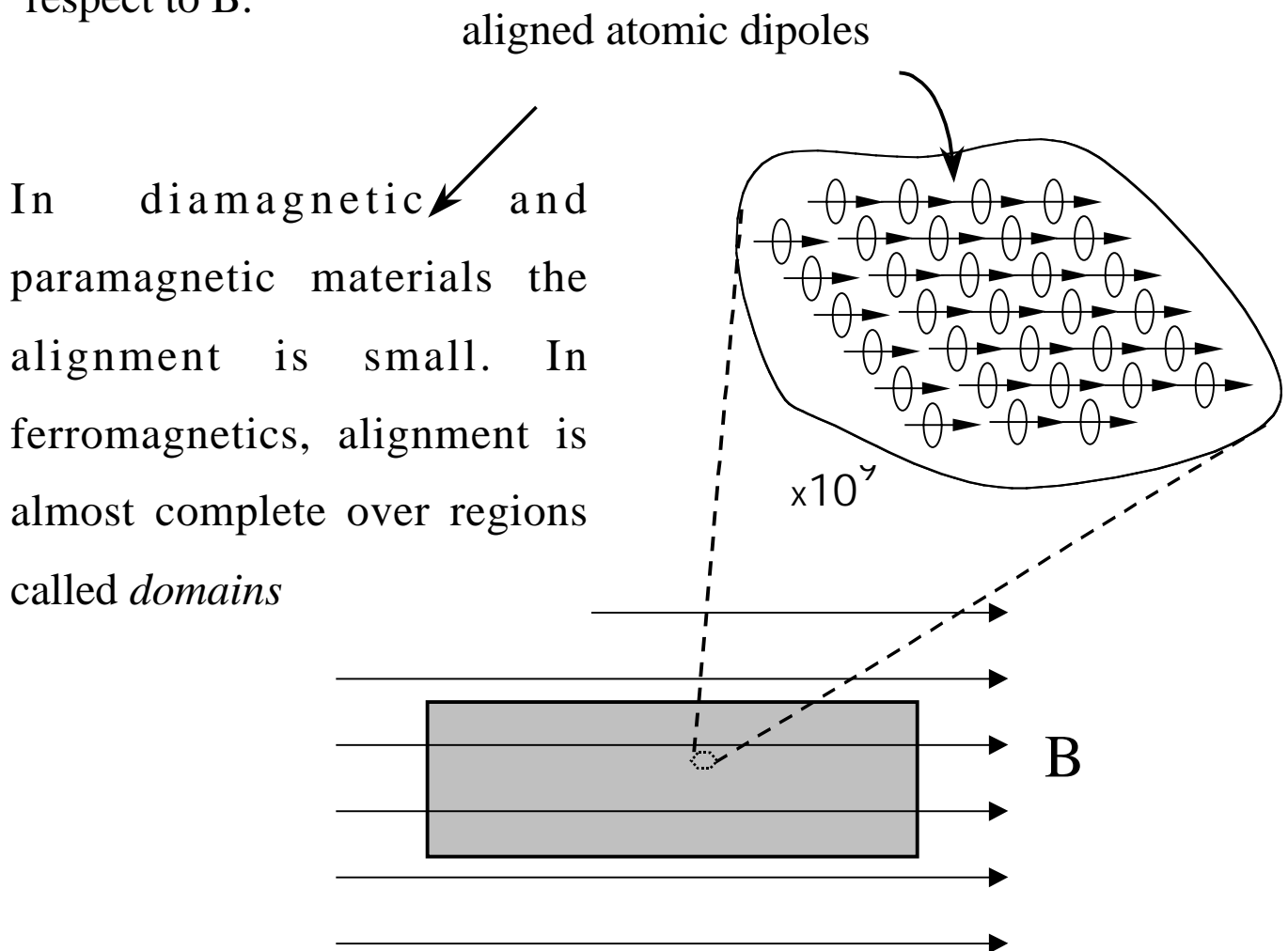
So far we have limited our discussion to the effects currents in conductors that generate fields in free space (where the permeability is $\mu_0=4\pi\times 10^{-7}$). But the permeability is not (generally) μ_0 in materials.

In matter, the electrons in motion in atomic orbitals may be viewed as an assembly of microscopic current loops – these are nanometre scale (10^{-9} m) current loops.

A very reasonable starting point for a theory of the magnetic properties of materials considers a bunch of nano-scale magnetic dipoles.

6.2 Broad classification of type of magnetic behaviour

If we put matter in a magnetic field B the magnetic dipoles (formed of the atomic orbitals due to circulating electrons) *tend* to align with respect to B .



The macroscopic, measurable property of a material due to its internal, microscopic dipole arrangement is called the **magnetization** of the material. Magnetization has the vector symbol \mathbf{M} and measures the dipole moment of the material.

Definition of **M**:

Magnetization **M** is the magnetic dipole moment *per unit volume*

M is measured in units amperes per metre, Am^{-1}

Hence, when a material is put in a magnetic field **B** we say it is **magnetized**.

If each atom has magnetic dipole moment **m** and there are **N** atoms per unit volume we have

$$\mathbf{M} = N\mathbf{m}$$

At the atomic level, the number of electrons per atom and the electronic configuration ($1s^2$, $2p^6$, $2s...$ etc) of elements differs – consider e.g. gold, aluminium, iron, copper.

Not surprisingly, the different electronic configurations lead to different types of magnetic behaviour.

Perhaps surprising is that the magnetic behaviour can be classified broadly as being one of the three:

Paramagnetic: magnetization \mathbf{M} is parallel to \mathbf{B}

Diamagnetic: magnetization \mathbf{M} is anti-parallel to \mathbf{B}

Ferromagnetic: magnetization $\mathbf{M} \neq 0$ after \mathbf{B} is removed

Thus, substances are classified **paramagnets**, **diamagnets** and **ferromagnets**, respectively.

Ferromagnetism is a *strong* magnetic effect: permanent magnets exhibit ferromagnetism – they can drive loudspeakers, start car engines, etc.

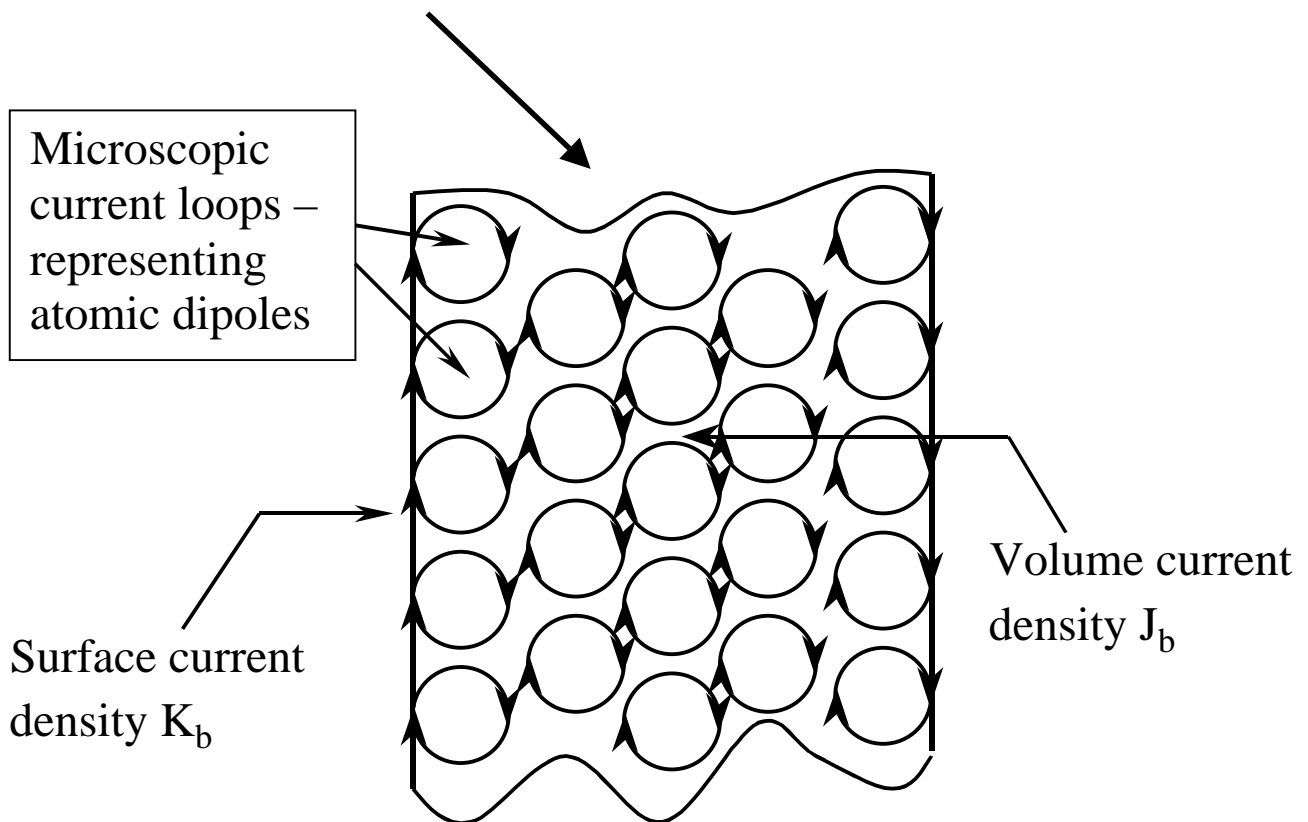
Effects of **paramagnetism** and **diamagnetism** are usually too weak to be obvious (i.e. not easily observable, unless the \mathbf{B} is very intense). In a non-uniform \mathbf{B} paramagnetic objects are attracted into the field, diamagnetic objects are repelled out.

6.3 Bound currents

The magnetic field produced by a magnetized object can be shown to be equivalent to that of the field generated by (bound) surface and volume currents in that object.

[Griffiths p263-266 shows how these fields are calculated from the vector potential $\mathbf{A}(\mathbf{r})$. We will follow a more ‘intuitive’ approach.]

Schematic cross-section of an element of magnetized material. \mathbf{M} is directed into the page



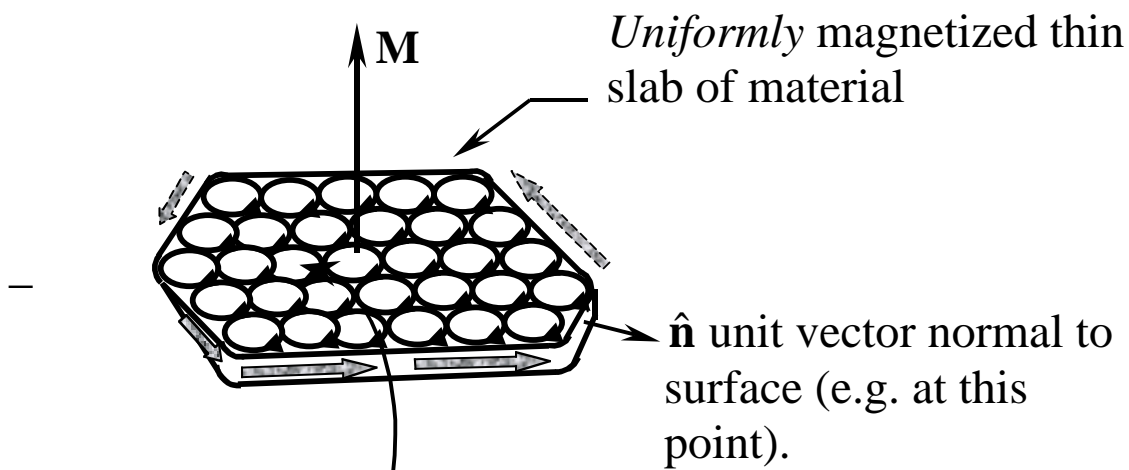
The surface and volume current densities are called **equivalent** or **Amperian** current densities and are:

$$\mathbf{K}_b = \mathbf{M} \times \hat{\mathbf{n}} \quad \text{surface current density}$$

\nwarrow
 unit vector normal to surface

$$\mathbf{J}_b = \nabla \times \mathbf{M} \quad \text{volume current density}$$

Uniform and non-uniform magnetization:



Interior, volume currents cancel leaving only the surface current running around the edge of the slab, as shown by shaded arrows.

Note the surface current direction is given by $\mathbf{M} \times \hat{\mathbf{n}}$

On the top and bottom surfaces $\mathbf{M} \parallel \hat{\mathbf{n}}$ gives $\mathbf{M} \times \hat{\mathbf{n}} = 0$, i.e. $\mathbf{K}_b = 0$

Note this is a *bound* current because each contributing current element is bound to an atom of the material making up the slab. The overall effect of the bound current is to produce the net observed surface current and the magnetization \mathbf{M} .

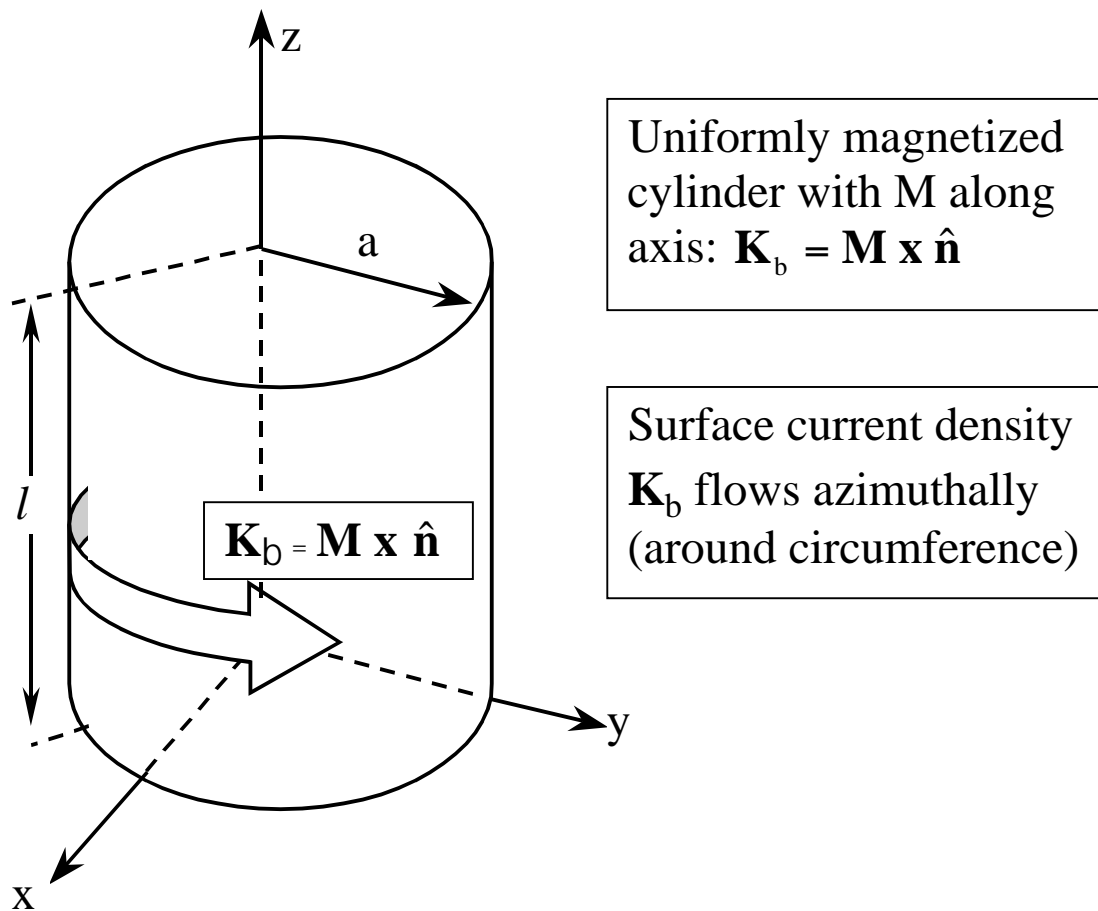
Non-uniform magnetization: if the magnetization of the object or specimen is non-uniform complete cancellation of the interior (volume) currents does not occur. In this case we will have

$$\mathbf{J}_b = \nabla \mathbf{x} \mathbf{M} \neq 0$$

(see pages 266-268 of Griffiths for a discussion)

6.3.1 Cylindrical bar magnet

Determine \mathbf{B} on the axis of the cylindrical magnet shown below



Inside the cylinder the magnetization \mathbf{M} is constant (uniform) so that

$$\mathbf{J}_b = \nabla \times \mathbf{M} = 0$$

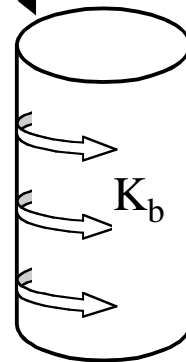
i.e. *no* bound volume current density. The bound surface current density is

$$\mathbf{K}_b = \mathbf{M} \times \hat{\mathbf{n}} = M_0 \hat{\mathbf{z}} \times \hat{\mathbf{r}}$$

We work in cylindrical coordinates r, ϕ, z and $\hat{\mathbf{z}} \times \hat{\mathbf{r}} = \hat{\phi}$ so

$$\mathbf{K}_b = M_0 \hat{\phi}$$

This describes a uniform current sheet around the circumference ($\hat{\phi}$ direction) of the cylinder

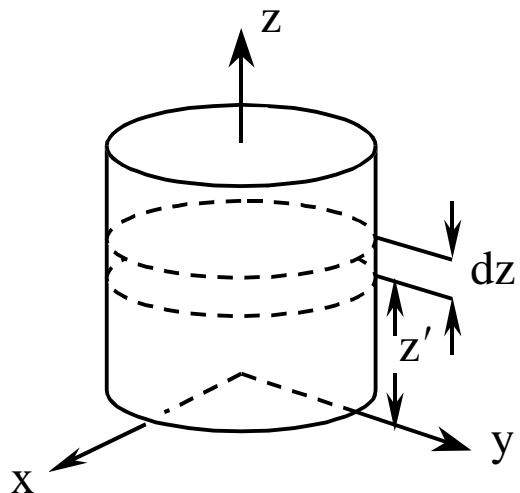


Note: there is no surface current on the cylinder's end faces.

To find \mathbf{B} on axis we use the expression for \mathbf{B} for a current loop

$$d\mathbf{B} = \frac{\mu_0 I a^2}{2(z^2 + a^2)^{3/2}} \hat{\mathbf{z}}$$

and integrate along z :



$$d\mathbf{B} = \frac{\mu_0 I a^2}{2(z^2 + a^2)^{3/2}} \hat{\mathbf{z}} \quad \Rightarrow \quad d\mathbf{B} = \frac{\mu_0 M_0 a^2 dz'}{2[(z - z')^2 + a^2]^{3/2}} \hat{\mathbf{z}}$$

$$\mathbf{B} = \int d\mathbf{B} = \int_0^l \frac{\mu_0 M_0 a^2 dz'}{2[(z - z')^2 + a^2]^{3/2}} \hat{\mathbf{z}}$$

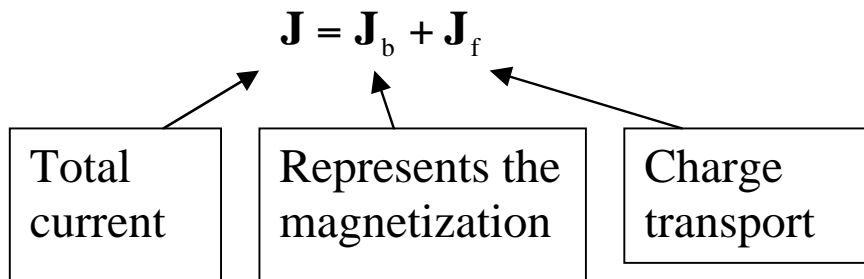
$$= \frac{\mu_0 M_0}{2} \left[\frac{z}{\sqrt{z^2 + a^2}} - \frac{z - l}{\sqrt{(z - l)^2 + a^2}} \right] \hat{\mathbf{z}}$$

6.4 Ampere's law including magnetic materials: $\mathbf{J} = \mathbf{J}_b + \mathbf{J}_f$

We have just seen that magnetization can be formulated in terms of bound currents (\mathbf{J}_b and \mathbf{K}_b).

In general, the problem may include bound currents (representing magnetization) *and* free currents, \mathbf{J}_f , due to conduction of a regular (ohmic) current through the material itself, for example.

In this general case we write the total current:



and Ampère's law becomes

$$\begin{aligned}\nabla \times \mathbf{B} &= \mu_0 \mathbf{J} = \mu_0 (\mathbf{J}_f + \mathbf{J}_b) \\ &= \mu_0 (\mathbf{J}_f + (\nabla \times \mathbf{M}))\end{aligned}$$

$\mathbf{J}_b = \nabla \times \mathbf{M}$

or,

$$\nabla \times \left(\frac{\mathbf{B}}{\mu_0} - \mathbf{M} \right) = \mathbf{J}_f$$

and we write

$$\nabla \times \mathbf{H} = \mathbf{J}_f$$

with

$$\mathbf{H} = \frac{1}{\mu_0} \mathbf{B} - \mathbf{M}$$

\mathbf{H} is called by a variety of names: the magnetic field intensity, the auxilliary field, ‘H’ or just ‘the magnetic field’.

(**N.B. caution:** ‘we’, including your lecturer, often speak loosely about ‘magnetic field’ meaning sometimes \mathbf{B} and sometimes \mathbf{H} – they are not the same!!)

Magnetization \mathbf{M} and magnetic field \mathbf{H} have units Am^{-1} .

6.5 Ampere’s law for \mathbf{H}

Looking at
$$\nabla \times \left(\frac{\mathbf{B}}{\mu_0} - \mathbf{M} \right) = \mathbf{J}_f$$

we see that Ampere’s law in terms of \mathbf{H} is

$$\nabla \times \mathbf{H} = \mathbf{J}_f$$

and,

Ampère’s law for \mathbf{H} ,
differential form

$$\oint \mathbf{H} \cdot d\mathbf{l} = I_{f,enc}$$

Ampère's law for \mathbf{H} ,
integral form

total free current enclosed
by Amperian loop

6.6 Why \mathbf{B} and \mathbf{H} ?

There is a correspondence, again, between electrostatics and magnetostatics. We have:

MAGNETIC

B H

ELECTRIC

E D

and the electric and magnetic quantities have correspondence:

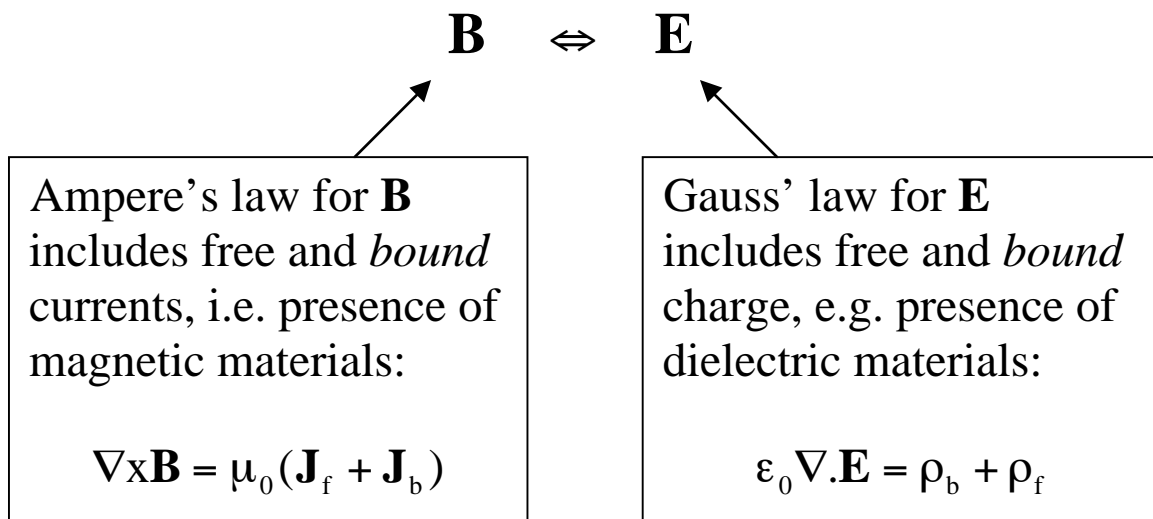
H \Leftrightarrow D

Gives us Ampere's
law in terms of free
current \mathbf{J}_f :

$$\nabla \times \mathbf{H} = \mathbf{J}_f$$

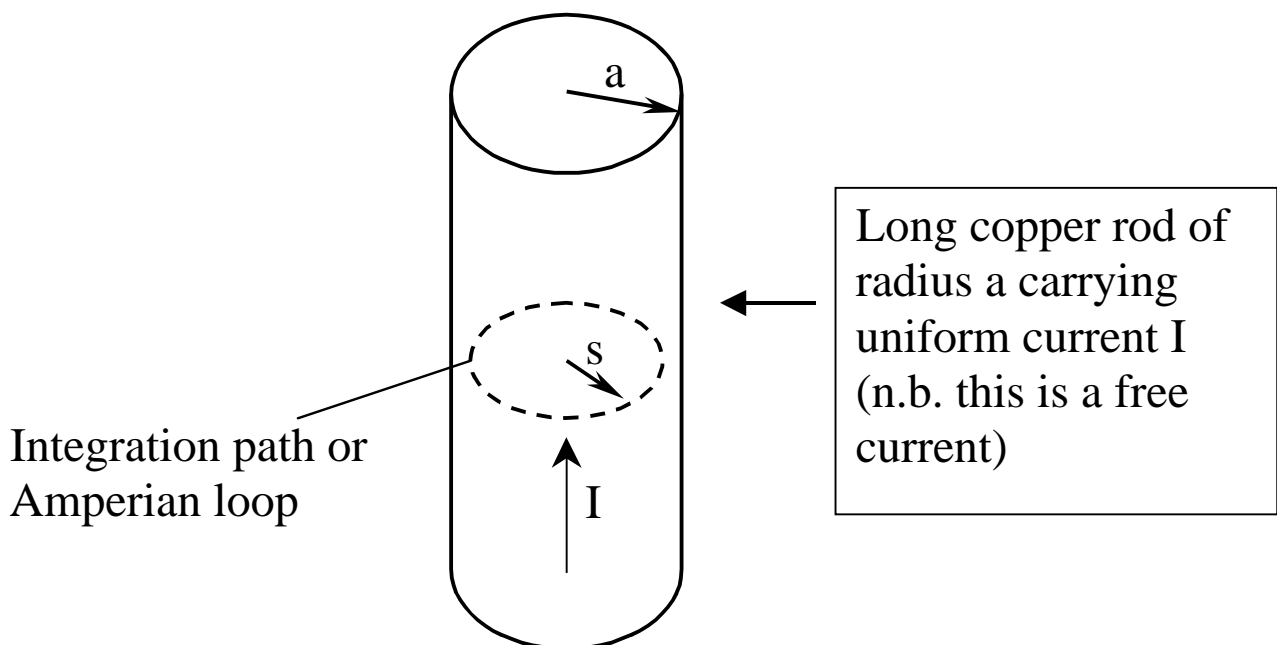
Gives us Gauss' law
in terms of the free
charge ρ_f :

$$\nabla \cdot \mathbf{D} = \rho_f$$

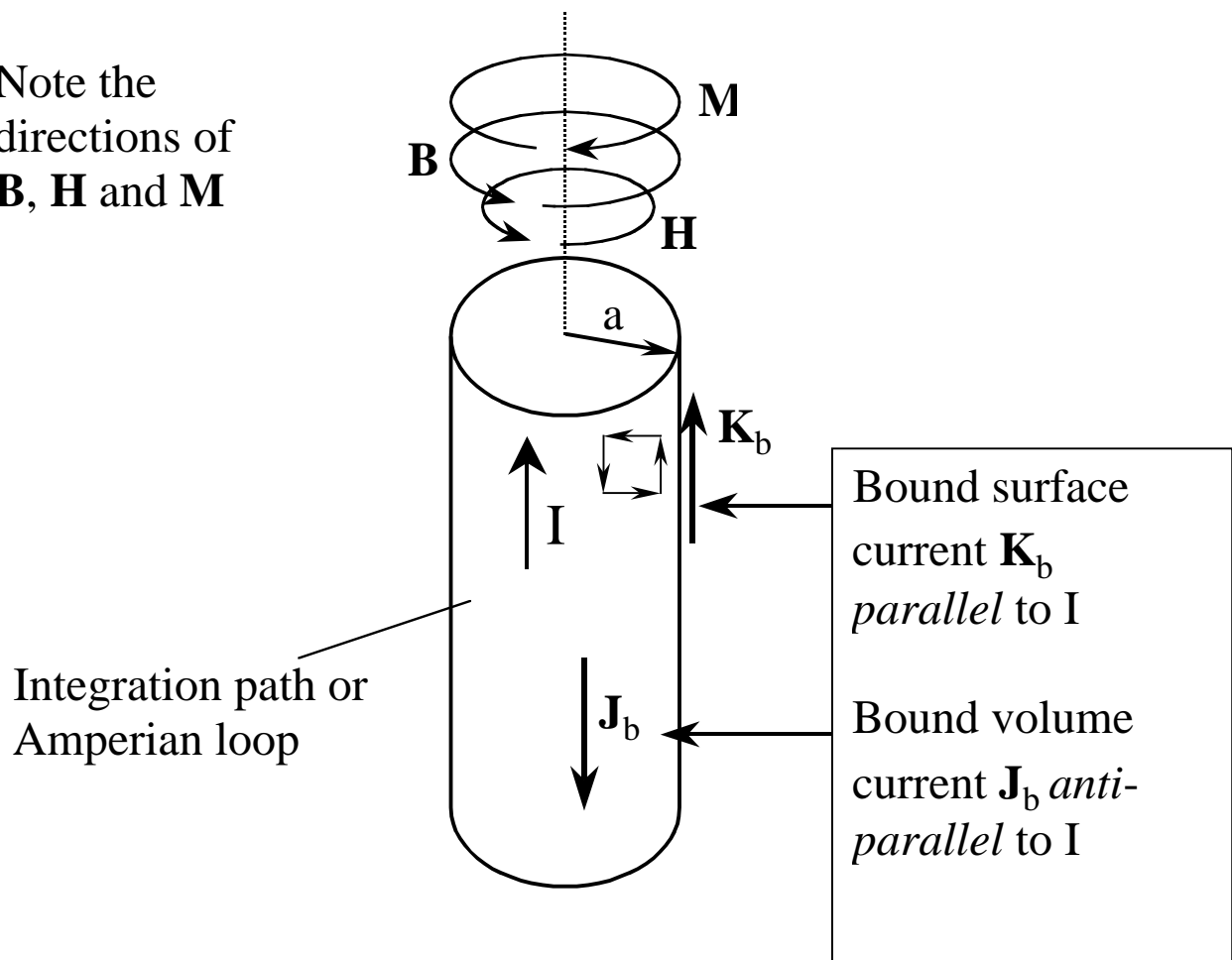


(See discussion on p271 of Griffiths where he explains the relative familiarity with **H** compared to **D**, which arises for practical reasons, even though ‘theoretically, they’re all on an equal footing’)

6.7 **H** for a current-carrying copper rod



Note the directions of \mathbf{B} , \mathbf{H} and \mathbf{M}



Copper is **diamagnetic** thus the atomic moments align antiparallel to \mathbf{B} and \mathbf{H} – but the magnetization is weak. These aligned moments correspond to bound surface (\mathbf{K}_b) and volume currents (\mathbf{J}_b) parallel to the axis: \mathbf{K}_b runs along the cylindrical surface parallel to the long axis. \mathbf{J}_b is a volume current antiparallel to the field.

Since the free and bound currents are longitudinal, \mathbf{M} , \mathbf{B} and \mathbf{H} are azimuthal (circumferential).

Ampere's law applied to the path of radius s gives

$$H(2\pi s) = I_{f,enc} = I \frac{\pi s^2}{\pi a^2}$$

so that

$$\mathbf{H} = \frac{I}{2\pi a^2} s \hat{\phi} \quad \text{inside the rod } (s \leq a)$$

and

$$\mathbf{H} = \frac{I}{2\pi s} \hat{\phi} \quad \text{outside the rod } (s \geq a)$$

Outside the wire we are in a region of free space and $\mathbf{M} = 0$. So that

$$\mathbf{H} = \frac{1}{\mu_0} \mathbf{B} - \mathbf{M} \quad \Rightarrow \quad \mathbf{B} = \mu_0 \mathbf{H}$$

\downarrow
 $\mathbf{M} = 0 \text{ for } s > a$

$$\mathbf{B} = \mu_0 \mathbf{H} = \frac{\mu_0 I}{2\pi s} \hat{\phi} \quad \text{outside the rod}$$

To find \mathbf{B} *inside* the rod we need to know (even though it is small in this case) \mathbf{M} .

6.8 Magnetic Susceptibility

When a magnetic field is applied to a material there is a dipole alignment leading to a non-zero magnetization. For **diamagnetic** and **paramagnetic** substances this magnetization is sustained only whilst there is a field applied.

For **linear media** we write,

$$\mathbf{M} = \chi_m \mathbf{H}$$

\uparrow
 χ_m is the **magnetic susceptibility**

χ_m is positive for paramagnetic materials

χ_m is negative for diamagnetic materials

e.g. Copper	$\chi_m = -9.7 \times 10^{-6}$	} diamagnets
Gold	$\chi_m = -3.4 \times 10^{-5}$	
Water	$\chi_m = -9.0 \times 10^{-6}$	

Aluminium	$\chi_m = 2.1 \times 10^{-5}$	} paramagnets
Gadolinium	$\chi_m = 4.8 \times 10^{-1}$	
Oxygen (gas)	$\chi_m = 1.9 \times 10^{-6}$	
Oxygen (liq)	$\chi_m = 3.9 \times 10^{-3}$	

Since $\chi_m = \frac{\mathbf{M}}{\mathbf{H}}$ (both in Am^{-1}) the magnetic susceptibility is a dimensionless quantity.

6.9 Magnetic Permeability

Recalling that

$$\mathbf{H} = \frac{1}{\mu_0} \mathbf{B} - \mathbf{M}$$

(Section 6.4) we see

$$\begin{aligned} \mathbf{B} &= \mu_0 (\mathbf{H} + \mathbf{M}) \\ &= \mu_0 (1 + \chi_m) \mathbf{H} \\ &= \mu \mathbf{H} \end{aligned}$$



$\mu = \mu_0 (1 + \chi_m)$ is the permeability

In free space (vacuum),

$$\mu = \mu_0(1 + \chi_m) = \mu_0$$

$$\chi_m = 0 \text{ in vacuo}$$

i.e. the permeability of free space is μ_0 – as we already knew!

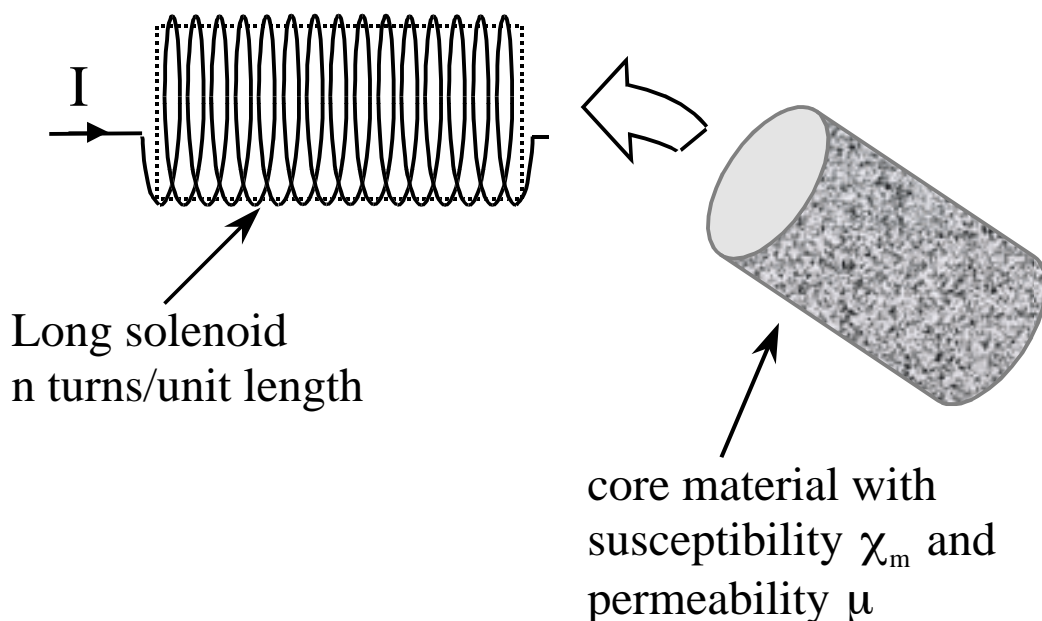
The **relative permeability** μ_r is just

$$\mu_r = \frac{\mu}{\mu_0} = (1 + \chi_m)$$

$$\mu_r = (1 + \chi_m)$$

relative permeability

6.10 Solenoid with a core

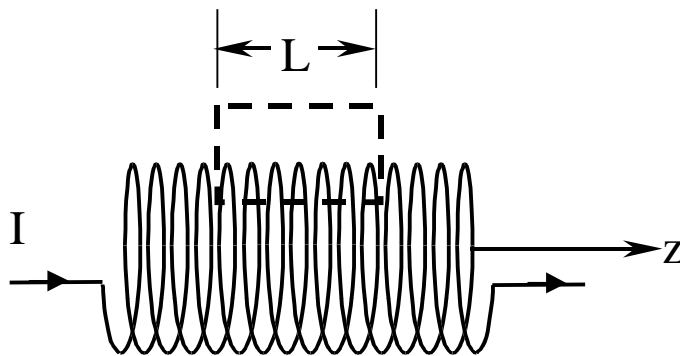


The magnetic induction \mathbf{B} inside the solenoid has contributions from free currents, I and equivalent bound current densities, \mathbf{K}_b and \mathbf{J}_b (arising from the magnetization of the core material).

Find \mathbf{H} first : we know Ampere's law for \mathbf{H} (Section 6.5)

$$\oint \mathbf{H} \cdot d\mathbf{l} = I_{f,enc}$$

free current enclosed by Amperian loop



$$HL = nIL$$

$$\mathbf{H} = nI \hat{\mathbf{z}}$$

and

$$\mathbf{B} = \mu \mathbf{H}$$

$$= \mu_0 (1 + \chi_m) \mathbf{H}$$

$$= \mu_0 (1 + \chi_m) nI \hat{\mathbf{z}}$$

Paramagnetic core:
 χ_m is positive (but
 usually small) \Rightarrow
 slight enhancement
 of \mathbf{B}

Diamagnetic core:
 χ_m is negative (but
 usually small) \Rightarrow
 slight decrease of \mathbf{B}

Ferromagnetic
 core: \Rightarrow large
 enhancement of \mathbf{B}
 since $\mu_r \sim 500$ or
 greater...

...but, caution:
 ferromagnetic
 properties are
 complex. (See
 next section.)