the energy supply for the emf that drives these currents, but it is certain that the rotation of the Earth plays a crucial role in the generation of the currents. These currents create a magnetic field which, at large distances from the core, is nearly a dipole field. The magnetic moment of the Earth is $8.0 \times 10^{22}\ \mathrm{A \cdot m^2}$. Figure 30.18 shows the magnetic field lines. Note that the field lines emerge from the surface of the Earth near the geographic South Pole and they reenter the surface near the geographic North Pole. The force experienced by a compass needle is due to this magnetic field.

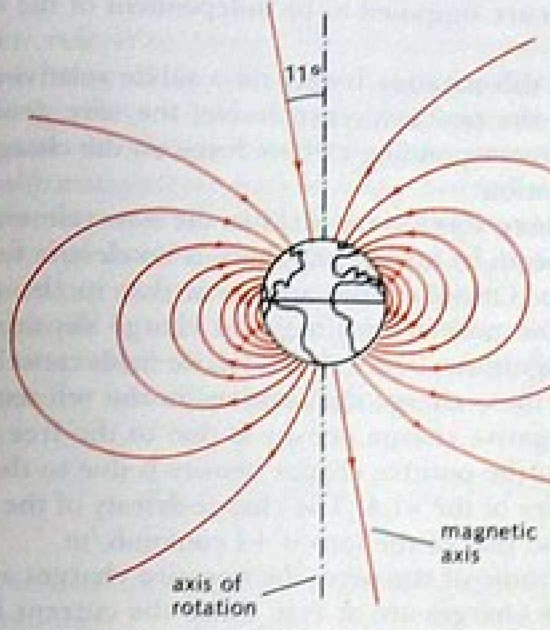

Fig. 30.18 Magnetic field of the Earth. The axis of the magnetic dipole makes an angle of 11° with the axis of rotation of the Earth.

30.4 Relativity and the Magnetic Field[4]

Since the magnetic force, $\mathbf{F} = q\mathbf{v} \times \mathbf{B}$, on a particle depends on the velocity, this force must necessarily depend on the reference frame with respect to which this velocity is reckoned. The following example illustrates this dependence in a drastic way.

Suppose that a positive charge q moves in the magnetic field generated by a current on a straight wire. We will assume that the (instantaneous) velocity $\mathbf{v}$ of the charge q is parallel to the wire. Figure 30.19 shows the situation in a reference frame in which the wire is at rest. The charge has a velocity $\mathbf{v}$ toward the right and the wire carries a current I toward the left. In this reference frame the current generates a magnetic field [see Eq. (32)]

$$B = \frac{\mu_0}{2\pi}\frac{I}{y} \tag{45}$$

and this magnetic field exerts a magnetic force

$$F = qvB = \frac{\mu_0}{2\pi}\frac{qvI}{y} \tag{46}$$

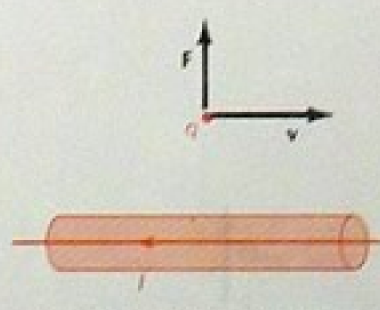

Fig. 30.19 In the rest frame of the wire, the charge q moves to the right with velocity $\mathbf{v}$.

[4] This section is optional.

on the charge q. The force points radially away from the wire (Figure 30.19).

Next, let us examine the situation in a new reference frame in which the charge q is (instantaneously) at rest. Relative to the old reference frame, this new reference frame moves toward the right with velocity **v**. Figure 30.20 shows the situation in this reference frame. The charge q is at rest and the wire moves toward the left with velocity −**v**. In this new reference frame there can be no magnetic force on the charge q, since the velocity of the charge is zero. We are now faced with a paradox: in the old reference frame the charge experiences a magnetic force and hence an acceleration; in the new reference frame the charge experiences no magnetic force, hence no acceleration — and yet accelerations are supposed to be independent of the reference frame!

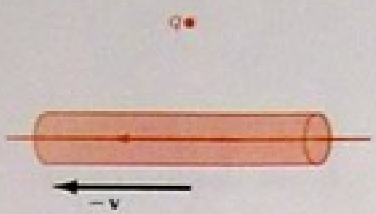

Fig. 30.20 In the rest frame of the charge q, the wire moves to the left with velocity −**v**.

The resolution of this paradox hinges on a subtle relativistic effect. It turns out that in the new reference frame, the wire generates an *electric field* and the corresponding electric force on the charge gives it the required acceleration.

To understand where this electric field in the new reference frame comes from, let us begin by asking why there is no electric field in the old reference frame. Obviously the answer is that in the reference frame of the wire the positive and negative charge densities on the wire are of equal magnitude; hence their electric fields cancel exactly.[5] Figure 30.21 shows these charge distributions in the reference frame of the wire. The negative charge density is due to the free electrons carrying the current; the positive charge density is due to the positive ions fixed in the lattice of the wire. The charge density of the electrons is $-\lambda$ coulomb/m and that of the ions is $+\lambda$ coulomb/m.

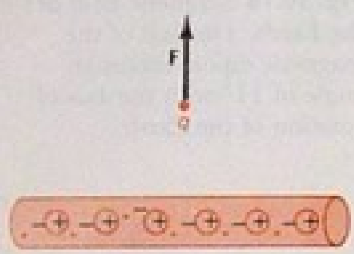

Fig. 30.21 In the reference frame of the wire, the positive and the negative charge densities on the wire are equal.

In the reference frame of the wire, the negative charges are in motion and the positive charges are at rest. Since the current is flowing toward the left, the free electrons carrying this current must move toward the right. For the sake of simplicity, let us assume that the velocity of the free electrons coincides with the velocity of the charge q. This is a very special case — and not very likely to happen in reality. It is, of course, possible to solve the problem in general, but it greatly helps in the solution if we have to worry about only a single velocity rather than two different velocities.

Now consider the electric charge densities in the new reference frame. The crucial point is that in this reference frame the negative and positive densities will *not be equal*. The inequality arises from the length contraction effect of special relativity. We recall from Section 17.5 that if an object has a certain length in its own reference frame, then in any other reference frame the length is shorter by a factor

$$\sqrt{1 - v^2/c^2}$$

where c is the speed of light. Thus, if a given number of positive charges sitting on the wire occupy a length of 1 m in the old reference frame (rest frame of the wire), they will occupy a shorter length of

$$1 \text{ m} \times \sqrt{1 - v^2/c^2}$$

[5] For the present purposes, we ignore the small electric field needed to push the current along the wire.

in the new reference frame. Correspondingly, the density of these positive charges will be larger: if the density is λ coulomb/m in the old reference frame, it will be

$$\lambda/\sqrt{1-v^2/c^2} \text{ coulomb/m}$$

in the new reference frame. For the negative charge distribution, the length contraction has the opposite effect — the density of negative charges will be smaller: if the density of charge is $-\lambda$ coulomb/m in the old reference frame, it will be

$$-\lambda\sqrt{1-v^2/c^2} \text{ coulomb/m}$$

in the new reference frame. This is so because in the old reference frame (rest frame of the wire) the charge distribution of electrons is in motion; it therefore is a *contracted* charge distribution. In the new reference frame, the charge distribution is at rest and it is not contracted. The transformation from the old to the new reference frame therefore is a transformation from a reference frame in which the length of the negative charge distribution is already contracted to a reference frame in which it is not contracted. Correspondingly, the density of the negative charges will be smaller, as indicated above.

In the new reference frame, the net charge per unit length of the wire is then the sum of *unequal* positive and negative contributions,

$$\lambda_{\text{new}} = \frac{\lambda}{\sqrt{1-v^2/c^2}} - \lambda\sqrt{1-v^2/c^2} \tag{47}$$

If the speeds are small compared to the speed of light, we can use the approximations

$$1/\sqrt{1-v^2/c^2} \cong 1 + \tfrac{1}{2}v^2/c^2 \quad \text{and} \quad \sqrt{1-v^2/c^2} \cong 1 - \tfrac{1}{2}v^2/c^2$$

so that Eq. (47) becomes

$$\lambda_{\text{new}} = \frac{\lambda}{\sqrt{1-v^2/c^2}} - \lambda\sqrt{1-v^2/c^2} \cong \lambda v^2/c^2 \tag{48}$$

Such a charge density along the wire will generate a radial electric field [see Eq. (24.10)]

$$E = \frac{1}{2\pi\varepsilon_0}\frac{\lambda_{\text{new}}}{y} = \frac{1}{2\pi\varepsilon_0 c^2}\frac{\lambda v^2}{y} \tag{49}$$

This electric field exerts an electric force

$$F_{\text{new}} = \frac{1}{2\pi\varepsilon_0 c^2}\frac{q\lambda v^2}{y} \tag{50}$$

on the charge q. This force points away from the wire (Figure 30.22).

In order to compare this electric force in the new reference frame with the magnetic force in the old reference frame, we note that the product of the velocity v of the electrons and their charge density λ is the current I on the wire. Hence Eq. (50) can be written

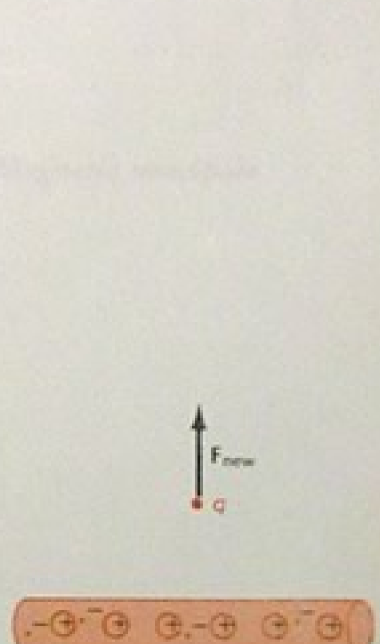

Fig. 30.22 In the reference frame of the charge q, the positive charge density on the wire exceeds the negative charge density.

$$F_{new} = \frac{1}{2\pi\varepsilon_0 c^2}\frac{qvI}{y} \tag{51}$$

Now examine the ratio of the forces in the old and the new reference frames,

$$\frac{F}{F_{new}} = \left(\frac{\mu_0}{2\pi}\frac{qvI}{y}\right) \Big/ \left(\frac{1}{2\pi\varepsilon_0 c^2}\frac{qvI}{y}\right) = \mu_0\varepsilon_0 c^2 \tag{52}$$

Inserting numerical values for the constants on the right side of Eq. (52), we find

$$\frac{F}{F_{new}} = 1.26 \times 10^{-6}\,\frac{\mathrm{N \cdot s^2}}{\mathrm{C^2}} \times 8.85 \times 10^{-12}\,\frac{\mathrm{C^2}}{\mathrm{N \cdot m^2}} \times (3.0 \times 10^8\ \mathrm{m/s})^2$$

$$= 1.0 \tag{53}$$

that is, the forces are exactly equal. Note that this result hinges on the fact that the values of ε_0 and μ_0 are exactly right to cancel the factor of c^2 in Eq. (52); again, as in Section 30.1, we see that there is a deep connection between electricity, magnetism, and the speed of light.

What we conclude from the above calculation is then the following: the force that the charges on the wire exert on the positive point charge q is the same in both the old and the new reference frames. The transformation of reference frame does not change the magnitude or direction of the force (and of the acceleration) — it only changes the character of the force from purely magnetic to purely electric. Incidentally, in a reference frame moving toward the right with a speed of, say, $\frac{1}{2}v$, the force would still have the same magnitude and direction, but it would be partially magnetic and partially electric.

Although we obtained these results only for a very special and simple case, the main features have general validity. If a particle experiences a magnetic force in a given reference frame, a transformation to the rest frame of the particle will make this magnetic force disappear. But in the latter reference frame, charge distributions will appear at the locations of the currents, and the electric force of these charge distributions will replace the original magnetic force. The net force is the same in both reference frames.[6]

Electric and magnetic forces and fields transform into one another if we change the frame of reference. In the rest frame of a charged particle, only electric forces act on the particle. Hence we can regard the magnetic forces that act on the particle in any other reference frame as resulting from a transformation of the electric forces in the rest frame. In this sense, magnetic forces can be regarded as a consequence of electric forces and of relativity.

[6] This invariance of the force is true only if the relative speed of the reference frames is low ($v \ll c$). If the speed is high, then we must take into account the relativistic transformation law for force. It turns out that the transformation of electric and magnetic forces is consistent with that law.