Spin and Relativity

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Abstract: The quantum relationship $m_0 c^2 = \hbar |\omega_0|$ may be regarded as the equivalence between two expressions for the rest energy of the particle, if ω_0 is considered as the spin angular velocity of the particle in its rest frame. The invariance of the relativistic space-time interval $ds^2 = c^2 dt^2 - dr^2$ to such a spin motion (space isotropy) leads to the spin momentum $S_z = \hbar/2$ for all structureless particles irrespective of their mass values. The inertia is an intrinsic property due to the spin motion of the particles, Mach's principle not being in this way necessary. The signs of the mass values occurring in the solutions of the Dirac equation might be related to the orientation of the spin motion, as suggested by the fundamental relationship $\pm m_0 c^2 = \pm|\omega_0| \hbar$.

**Keywords:** spin, relativity, Dirac equation, Mach's principle.

As we know, the spin cannot be motivated in the frame of classical mechanics. Even in the nonrelativistic quantum theory, the nature of spin remains unclear. The spin results solely from Dirac's equation.[1] Although the Pauli and Dirac matrices undoubtedly show the spin existence, there is some mystery as to the physical origins of and in the visualization of the spin.[2,3] One may says that spin is an intrinsic property of the matter. It must have to do with relativity even though this connection is not entirely understood.[4]

In this paper, I shall try to sketch a simple motivation for the existence of spin starting from the fundamental relationship, and namely:

$$m_0 c^2 = \hbar |\omega_0| \quad , \tag{1}$$

where m_0 is the rest mass and $m_0 c^2$ the rest energy of the particle.

If the particle is considered as being a physical object spinning with angular velocity ω_0 (Figure 1), the right-hand side of equation (1) should be regarded as another relativistic expression for the rest energy.
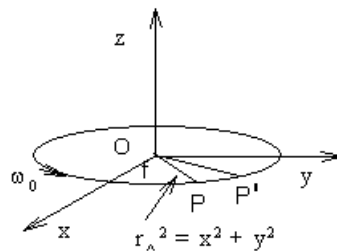
Figure 1. A spinning reference frame.

We may argue the existence of the spin motion in the space-time frame of the particle as follows: unlike space coordinates, time is not directly measurable (observable). The simplest way to estimate time is to consider uniform motion. One can obtain time by comparing covered distances (Aristotle[5]: time is the number of motion). This leads to the necessity of introducing motion in the space-time reference frame. The only allowed motion in the rest frame of the particle should be that of rotation (spin).

Therefore, it is reasonable to assume that the rest energy of a particle is related to its spin motion, which is only allowed in that system.[3] This reasoning allows us to regard $\hbar |\omega_0|$ as an equivalent expression for the rest energy of the particle.

The particle as a moving object must also obey another fundamental relationship, namely the relativistic elementary "space-time interval" between the physical events of the particle:

$$ds^2 = c^2 dt^2 - dr^2 \quad . \tag{2}$$

Every physical process, such as translation, rotation, etc., must be related to expression (2). The invariance of ds^2 to uniform translation (space homogeneity) leads to the Lorentz corrections.[6]

Let us now consider the uniform rotation (spin) of the reference frame, with angular velocity ω_0 in x,y-plane around the z-axis, as shown in Figure 1. In this case, we have

$$x \rightarrow x' \; ; \; y \rightarrow y' \; ; \; z=z' \; ; \; t \rightarrow t'= t+dt \; ; \; \omega_0 = d\phi/dt = const.$$

and ds^2 (equation 2) becomes

$$ds'^2 = (c^2 - r_\perp^2 \, |\omega_0|^2) \, dt^2 - 2 \, |\omega_0| \, (ydx - xdy) \, dt - dr^2 \, , \tag{3}$$

where $r_\perp^2 = x^2 + y^2$ represents the radius perpendicular to the rotation z-axis, i.e. the distance from origin to the points P, P', etc. (Figure 1).

Expression (3) can easily be derived.[7] Note that the linear velocity $v = r_\perp |\omega_0|$ must obey the restriction imposed by the special relativity, i.e. $r_\perp^2 |\omega_0|^2 < c^2$. For the limit case $r_\perp |\omega_0| = c$, the rotating space becomes 'closed' with the lateral radius $r_\perp$. This is all what Fock mentioned.[7]

But such a rotating empty space is physically meaningless. We must therefore actually ascribe this rotation to the particle situated in the origin of this space. For that particle, the set of points P, P', etc. for which $r_\perp |\omega_0| = c$, should be considered the closure (frontier) of the particle. We do not know too much about the shape of a spinning particle, considered as being structureless, but we can at least define for it a radial extension equal to:

$$r_\perp = c/|\omega_0| \quad . \tag{4}$$

From equations (1) and (4), we have:

$$r_\perp = \hbar /m_0 c = \lambda_C \; \text{(Compton radius)} \quad . \tag{5}$$

This result shows that all structureless particles with rest mass cannot be pointlike. For the limit case, $r_\perp |\omega_0| = c$, expression (3) becomes:

$$ds'^2 = 2 \, |\omega_0| \, (x.y/dt - y.dx/dt) dt^2 - dr^2 \quad . \tag{6}$$

We have already mentioned the invariance of ds^2 to the uniform translation (space homogeneity). Let us now consider the invariance of ds^2 to the uniform rotation (space isotropy). In other words, for a noninteracting spinning particle, space must remain unaffected by the uniform rotation. From the invariance condition ds'^2 (equation 6)$\equiv ds^2$ (equation 2), we obtain:

$$2\,|\omega_0|\,(x.dy/dt - y.dx/dt) = c^2 \quad . \tag{7}$$

If we use now the fundamental relationship from equation (1), we have

$$S_z = m_0|(x.dy/dt - y.dx/dt)| = m_0|r \wedge v|_z = \hbar/2 \quad , \tag{8}$$

where S_z is the classical expression for the z-component of angular momentum. The result is interesting. This mainly shows that the $\hbar/2$ value of the angular (spin) momentum preserves the space isotropy. It must be universal and characteristic for all structureless particles with finite rest mass, independent of their mass values.

If the two possible rotations around z-axis are considered ($\omega_0 \in \{+|\omega_0|; -|\omega_0|\}$) corresponding to $x \rightarrow y$ and $y \rightarrow x$ rotations, both the $\pm \hbar/2$ values conserve the space isotropy. The time reversal $t \rightarrow -t$ in (8) leads to $-\hbar/2$ value. Note that for $\pm|\omega_0|$ values equation (1) becomes $\pm|\omega_0|\hbar = \pm m_0 c^2$. The mass values $\pm m_0$ occurring in Dirac's equation might actually be related to the rotation sense of the spin angular velocity ω_0.

Moreover, according to (1) the rest mass m_0 is tightly connected with the spin motion represented by ω_0. Therefore, a structureless elementary particle with a finite rest mass and radial extension behaves as a small mechanical top, its inertial properties not necessarily being conditioned by the gravitational interaction with the matter in universe (Mach's principle).

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