GRAVITATIONAL AND INERTIAL MASS FLUCTUATIONS

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ABSTRACT: The purpose of this paper is to reveal a portion of classical physics that contains an intrinsic *relativistic* phenomenon called Gravitational Mass Fluctuations. A correlation has been established between mass, inductors, and capacitors, thereby linking gravity to electromagnetism. A simplified gravitational mass relativity model called Natural Relativity (NR) Theory is presented and shown to be a primary gravitational effect. This theory is correlated to Einstein's Special Relativity (SR) Theory, and as a consequence, creates a new "Principle of Equivalence Theorem" and a secondary gravitational effect. A temporal rotation operator is introduced using Euler's Identity, which shows the *complex* (i.e., time-future) motion of matter. The speed of light *c*, Planck's constant *h*, permeability μ_0, permittivity ε_0, Boltzmann's Constant *k*, electric charge *q*, and the Fine Structure Constant α are invariant between equipotential surfaces of gravity because the fluctuation or curvature of the parameters that compose these constants are shown to evaluate to unity gain. In other words, these constants remain constant anywhere with a given gravity well. Gravitomagnetic Theory shows that the magnetic field energy produced by a moving electron is equivalent to its' *special relativistic* mass fluctuation, and therefore, is shown to couple to gravity. This motion can either have a typical velocity or a *complex* (i.e., time-future) velocity. If the velocity is *complex,* then the *special relativistic* mass fluctuation of an electron is NEGATIVE, exhibit an antigravitational effect, and produce a *complex* (i.e., time-future) magnetic field. In addition, the total field energy of a *complex* magnetic field contained within a volume is NEGATIVE. In the Bohr model of the Hydrogen atom, an Amperian Current is described as an electron circulating around a nucleus at a *relativistic* speed. This creates a magnetic induction emerging from the center of the nucleus. Canceling this field by applying an external magnetic induction causes the velocity of the electron to become *complex*. The presence of NEGATIVE RESISTANCE, the production of NEGATIVE ENERGY, and the control of GRAVITY/ANTIGRAVITY occur by fluctuating the mass of an object. The theory presents a conceptual breakthrough in energy and high-speed field propulsion technology, and explores solutions based entirely within the framework of classical physics.

INTRODUCTION

Puthoff (1996) coined the phrase, "metric engineering", and Puthoff, Little and Ibison (2002) consider the vacuum to be a polarizable medium, and that it can be expressed in terms of tensor formulations of curved space-time. The bending of light passing near a massive object is caused by induced spatial variation in the refractive index of the vacuum near the object. This is correlated to changes in permeability μ_0 and permittivity ε_0 of the vacuum. Changes occurring in the vacuum also affect the mass of objects, the length and bending of rulers, the frequency of clocks, the energy of light, etc. This paper links gravity with electromagnetism by presenting formulations of curved space-time in terms of classical physics, which are caused by *relativistic* fluctuations of mass M, inductance L, and capacitance C of an object. For example, when an object with mass M naturally falls downward in a given gravity well, its' *natural relativistic* mass $\dot{M}$ increases due to Newtonian Gravitation, or universal mass attraction. Therefore, the new mass of an object is displaced to a new position within this well, and mass-energy remains conserved. However, by converting this increase in *relativistic* mass $\dot{M}$ to energy, a force acts upon the object, and the new mass is now displaced to its original position that was higher vertically in the well. The object exhibits an antigravitational effect. The rate of change of this fluctuation could cause the speed of the object to easily *exceed* the speed of light. This is because the *relativistic* gravitational mass of the object, which is shown to be convergent, is moving at right angles to a *relativistic* inertial mass, which is shown to be divergent. Since the speed of the object with *relativistic* gravitational mass has no known upper limit, the resulting speed through deep space could be enormous and necessitates the use of the warp factor equation.

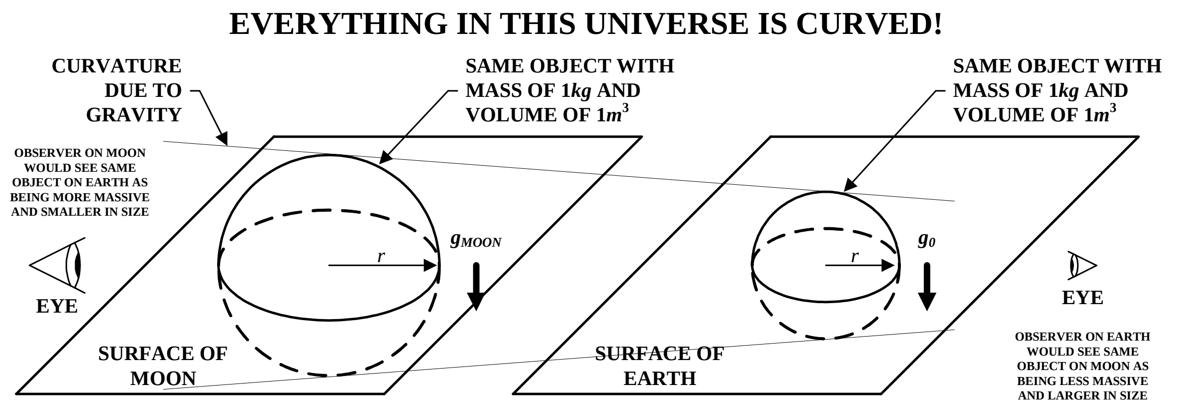


FIGURE 1. The same sphere changes in mass and size due to changes of gravity.

Shown above are two spheres with equal mass and size. Since the gravity of the Moon g_{MOON} is approximately $1/6$ the gravity of the Earth g_0, an observer on the Moon would measure an identical sphere on the Earth as having more mass and being smaller in volume. Likewise, an observer on the Earth would measure an identical sphere on the Moon as having less mass, and larger in volume. This is due to the curvature of space and time caused by universal mass attraction, or gravity. So, *relative* to an observer on the Earth, a $1kg$ sphere of mass on the Moon has less mass than the same $1kg$ sphere of mass on the Earth. And, a $1m^3$ sphere of volume on the Moon is larger in size than the same $1m^3$ sphere of volume on the Earth. This *relativistic* change in mass and volume are referred to as **Gravitational Mass Fluctuations**, or **GMF**.

THE MUTUAL EXCLUSION PRINCIPLE

Marmet (2001) considers "separately" the influence of a gravitational potential upon matter, and assumes for the moment that kinetic energy is zero. He's *implicitly* invoking what I call the *mutual exclusion principle*, and therefore, considers kinetic energy and gravitational energy independently. The *mutual exclusion principle* is a tool used to compute the curvature or fluctuation of various parameters related to **Gravitational Energy** systems. For **Kinetic Energy** systems, the following flux-based parameters mass M, inductor L, and capacitor C, are invariant between equipotential surfaces of gravity g_Y. However, for **Gravitational Energy** systems, and given an equipotential surface of gravity reference, the following temporal-based parameters *relativistic* mass $\pm\Delta M$, *relativistic* inductor $\pm\Delta L$, and *relativistic* capacitor $\pm\Delta C$, fluctuate or curve between equipotential surfaces of gravity. The kinetic energy of **Gravitational Energy** systems is assumed to be zero. By applying the product rule, the *mutual exclusion principle* is mathematically expressed as,

$$z(t) = \frac{d}{dt}(x\,y) = y\frac{dx}{dt} + x\frac{dy}{dt} = y\,\dot{x} + x\,\dot{y}$$

Where, the **Flux Coupling Term** is,

$$z(t) = y\,\dot{x}$$

And the **Gravitational Coupling Term** or temporal-based fluctuating system is,

$$z(t) = x\,\dot{y}$$

The first term is regarded as kinetic or fluxes, and therefore, couples to inertia and is Newtonian-based. The second term is regarded as temporal, and therefore, couples to gravity and is non-Newtonian-based. This principle, which has been *implicitly* used for several centuries, excludes one term from the other. The fixed distance $+y$ and the changing distance $+\dot{y}$ or $+\Delta y$ are directed *towards* the center of gravity. The fixed distance $+x$ and the changing distance $+\dot{x}$ or $+\Delta x$ are directed *across* an equipotential surface of gravity.

GRAVITATIONAL MASS FLUCTUATION

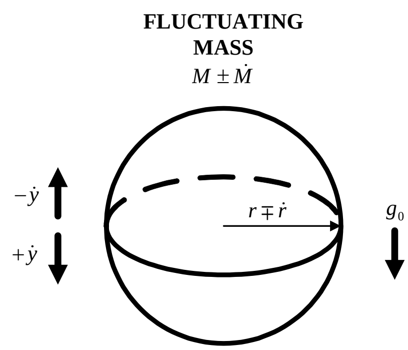


FIGURE 2. The fluctuating mass of an object.

The *complete* ideal momentum model is composed of two terms,

$$f_M(t) = \frac{dp_Y}{dt} = \frac{d(M\,v_Y)}{dt} = M\,\frac{dv_Y}{dt} + v_Y\,\frac{dM}{dt} = M\,\dot{v}_Y + v_Y\,\dot{M} \tag{1}$$

Where, the **Flux Coupling Term** is $M\,\dot{v}_Y$, and mass M is invariant within any equipotential surface of gravity g_Y. The **Gravitational Coupling Term** is $v_Y\,\dot{M}$, and changing mass $\dot{M}$ fluctuates between equipotential surfaces of gravity.

For a “mass fluctuating system”, the **Gravitational Coupling Term** is NOT zero Newtons. So, given an object having mass M moving at constant velocity v_Y, or $\dot{v}_Y = 0\,m/s^2$, the **Flux Coupling Term** is,

$$f_M(t) = M\,\dot{v}_Y = 0\,N \tag{2}$$

This removes the **Flux Coupling Term**, leaving only the **Gravitational Coupling Term**,

$$f_M(t) = v_Y\,\dot{M} \neq 0\,N \tag{3}$$

Since $\dot{M}$ has units of resistance in mNs/m^2, its direction of change could either be POSITIVE or NEGATIVE. If $\dot{M}$ is negative, it has units of negative resistance or,

$$\dot{M} < 0\,mNs/m^2 \tag{4}$$

Now, the instantaneous gravitationally induced power P_M of a fluctuating mass $\dot{M}$ is,

$$P_M(t) = v_Y f_M(t) = {v_Y}^2 \dot{M} \tag{5}$$

So, for certain values of $\dot{M}$, the total instantaneous power P_M can be NEGATIVE or,

$$P_M(t) < 0\,Watts \tag{6}$$

Then, integrating P_M with respect to time when the total power is less than zero watts results in NEGATIVE energy of mass M or,

$$E_M(t) = \int P_M(t)\,dt = {v_Y}^2 \int \dot{M}\,dt = {v_Y}^2\, M(t) < 0\,Joules \tag{7}$$

If $\dot{M}$ is positive, it has units of positive resistance or,

$$\dot{M} > 0\,mNs/m^2 \tag{8}$$

Now, the instantaneous gravitationally induced power P_M of a fluctuating mass $\dot{M}$ is,

$$P_M(t) = v_Y f_M(t) = {v_Y}^2\, \dot{M} \tag{9}$$

So, for certain values of $\dot{M}$, the total instantaneous power P_M can be POSITIVE or,

$$P_M(t) > 0\,Watts \tag{10}$$

Then, integrating P_M with respect to time results in excess POSITIVE energy of mass M or,

$$E_M(t) = \int P_M(t)\,dt = {v_Y}^2 \int \dot{M}\,dt = {v_Y}^2\, M(t) > 0\,Joules \tag{11}$$

So, the energy equivalent of mass (**gravitational energy**) E_M is,

$$E_M(t) = M(t)\,{v_Y}^2 \tag{12}$$

By rearranging terms, the mass equivalent of energy M is,

$$M(t) = \frac{E_M(t)}{{v_Y}^2} \tag{13}$$

The fluctuating mass equivalent of energy $\dot{M}$ is,

$$\dot{M} = \frac{\dot{E}_M}{{v_Y}^2} \tag{14}$$

So, the gravitational momentum model is,

$$f_M(t) = \dot{M}\,v_Y = \frac{\dot{E}_M}{{v_Y}^2} v_Y = \frac{\dot{E}_M}{v_Y} \tag{15}$$

Letting $\dot{y} = dy/dt = v_Y$, the **Gravitational Force Coupling Term** is,

$$f_M(t) = \frac{\dot{E}_M}{\dot{y}} \tag{16}$$

By rearranging terms, the fluctuating energy equivalent of mass $\dot{E}_M$ is,

$$\dot{E}_M = f_M(t)\,\dot{y} \tag{17}$$

The total energy E_M contained within matter is,

$$E_M(t) = M(t)\,c^2 \tag{18}$$

The change in this total energy $\dot{E}_M$ is,

$$\dot{E}_M = \dot{M}\,c^2 \tag{19}$$

By rearranging terms, the change in total mass $\dot{M}$ is,

$$\dot{M} = \frac{\dot{E}_M}{c^2} \tag{20}$$

So, the time derivative form of the **Gravitational Mass Coupling Term** is,

$$\dot{M} = \frac{f_M(t)\,\dot{y}}{c^2} \tag{21}$$

The derivative form is,

$$dM = \frac{f_M\,dy}{c^2} \tag{22}$$

The difference form is,

$$\Delta M = \frac{f_M\,\Delta y}{c^2} \tag{23}$$

GRAVITATIONAL INDUCTIVE MASS FLUCTUATION

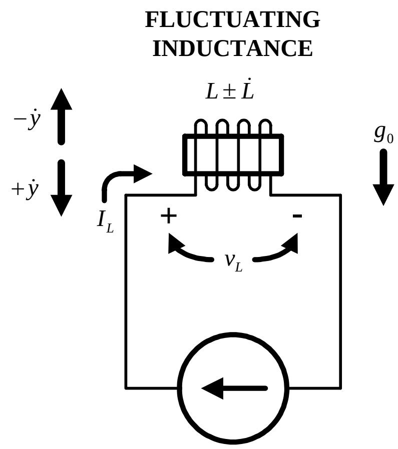


FIGURE 3. The fluctuating inductance of an object.

The *complete* ideal inductor model is composed of two terms,

$$v_L(t) = \frac{d}{dt}\left(L\,I_L\right) = L\frac{dI_L}{dt} + I_L\frac{dL}{dt} = L\,\dot{I}_L + I\,\dot{L}_L \tag{24}$$

Where, the **Flux Coupling Term** is $L\,\dot{I}_L$, and inductance L is invariant within any equipotential surface of gravity g_Y. The **Gravitational Coupling Term** is $I\,\dot{L}_L$, changing inductance $\dot{L}$ fluctuates between equipotential surfaces of gravity.

For an "inductive fluctuating system", the **Gravitational Coupling Term** is NOT zero volts. By applying a constant current I_L through inductor L, or $\dot{I}_L = 0\,Amps/s$, the **Flux Coupling Term** is,

$$v_L(t) = L\,\dot{I}_L = 0\,Volts \tag{25}$$

This removes the **Flux Coupling Term**, leaving only the **Gravitational Coupling Term,**

$$v_L(t) = I_L\dot{L} \neq 0\,Volts \tag{26}$$

Since $\dot{L}$ has units of resistance in ohms, Ω, its direction of change could either be POSITIVE or NEGATIVE. If $\dot{L}$ is negative, it has units of negative resistance or,

$$\dot{L} < 0\,\Omega \tag{27}$$

Now, the instantaneous gravitationally induced power P_L of a fluctuating inductor $\dot{L}$ is,

$$P_L(t) = I_L\,v_L(t) = I_L^{\,2}\,\dot{L} \tag{28}$$

So, for certain values of $\dot{L}$, the total instantaneous power P_L can be NEGATIVE or,

$$P_L(t) < 0\,Watts \tag{29}$$

Then, integrating P_L with respect to time when the total power is less than zero watts results in NEGATIVE energy of inductor L or,

$$E_L(t) = \int P_L\, dt = I_L{}^2 \int \dot{L}\, dt = I_L{}^2\, L(t) < 0\, Joules \tag{30}$$

If $\dot{L}$ is positive, it has units of positive resistance or,

$$\dot{L} > 0\,\Omega \tag{31}$$

Now, the instantaneous gravitationally induced power P_L of a fluctuating inductor $\dot{L}$ is,

$$P_L(t) = I_L\, v_L(t) = I_L{}^2\, \dot{L} \tag{32}$$

So, for certain values of $\dot{L}$, the total instantaneous power P_L can be POSITIVE or,

$$P_L(t) > 0\, Watts \tag{33}$$

Then, integrating P_L with respect to time results in excess POSITIVE energy of inductor L or,

$$E_L(t) = \int P_L\, dt = I_L{}^2 \int \dot{L}\, dt = I_L{}^2\, L(t) > 0\, Joules \tag{34}$$

Equate this to the energy equivalent of mass E_M,

$$E_L(t) = E_M(t) \tag{35}$$

Then, the energy equivalent of mass (**gravitational energy**) E_L is,

$$E_L(t) = v_Y{}^2\, M_L(t) = I_L{}^2\, L(t) \tag{36}$$

By rearranging terms, the mass equivalent of energy M_L is,

$$M_L(t) = \frac{E_L(t)}{v_Y{}^2} = \frac{I_L{}^2\, L(t)}{v_Y{}^2} \tag{37}$$

$$v_Y{}^2 = \frac{I_L{}^2\, L(t)}{M_L(t)} \tag{38}$$

The fluctuating mass equivalent of energy $\dot{M}_L$ is,

$$\dot{M}_L = \frac{\dot{E}_L}{v_Y{}^2} = \frac{I_L{}^2\, \dot{L}}{v_Y{}^2} \tag{39}$$

So, the gravitational inductor model is,

$$f_L(t) = \dot{M}_L\, v_Y = \frac{\dot{E}_L}{v_Y{}^2} v_Y = \frac{\dot{E}_L}{v_Y} = \frac{I_L{}^2 \dot{L}}{v_Y} \tag{40}$$

Letting $\dot{y} = dy/dt = v_Y$, the **Gravitational Inductive Force Coupling Term** is,

$$f_L(t) = \frac{\dot{E}_L(t)}{\dot{y}} \tag{41}$$

By rearranging terms, the fluctuating energy equivalent of mass $\dot{E}_L$ is,

$$\dot{E}_L = f_L(t)\,\dot{y} \tag{42}$$

The total energy E_L contained within matter is,

$$E_L(t) = M_L(t)\,c^2 \tag{43}$$

The change in this total energy $\dot{E}_L$ is,

$$\dot{E}_L = \dot{M}_L\,c^2 \tag{44}$$

By rearranging terms, the change in total mass $\dot{M}$ is,

$$\dot{M}_L = \frac{\dot{E}_L}{c^2} = \frac{f_L(t)\,\dot{y}}{c^2} = \frac{I_L{}^2\,\dot{L}}{\dot{y}^2} = \frac{I_L{}^2\,\dot{L}}{\dfrac{I_L{}^2\,L(t)}{M_L(t)}} = \frac{M_L(t)\,\dot{L}}{L(t)} \tag{45}$$

So, the time derivative form of the **Gravitational Inductive Coupling Term** is,

$$\dot{L} = \frac{f_L(t)\,L(t)\,\dot{y}}{M_L(t)\,c^2} \tag{46}$$

The derivative form is,

$$dL = \frac{f_L\,L\,dy}{M_L\,c^2} \tag{47}$$

The difference form is,

$$\Delta L = \frac{f_L\,L\,\Delta y}{M_L\,c^2} \tag{48}$$

GRAVITATIONAL CAPACITIVE MASS FLUCTUATION

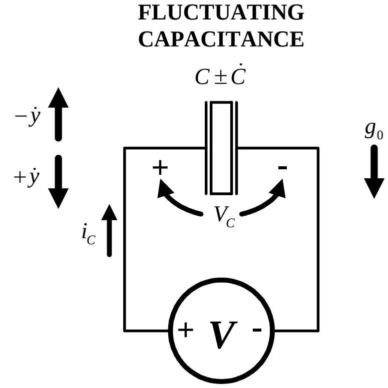


FIGURE 4. The fluctuating capacitance of an object.

The *complete* ideal capacitor model is composed of two terms,

$$i_C(t) = \frac{d}{dt}(C V_C) = C\frac{dV_C}{dt} + V_C\frac{dC}{dt} = C\dot{V}_C + V_C\dot{C} \tag{49}$$

Where, the **Flux Coupling Term** is $C\dot{V}_C$, and capacitor C is invariant within any equipotential surface of gravity g_Y. The **Gravitational Coupling Term** is $V_C\dot{C}$, and changing capacitance $\dot{C}$ fluctuates between equipotential surfaces of gravity.

For a "capacitive fluctuating system", the **Gravitational Coupling Term** is NOT zero amps. By applying a constant voltage across capacitor C, or $\dot{V}_C = 0 Volts/s$, the **Flux Coupling Term** is,

$$i_C(t) = C\dot{V}_C = 0\,Amps \tag{50}$$

This removes **Flux Coupling Term**, leaving only the **Gravitational Coupling Term,**

$$i_C(t) = V_C\dot{C} \neq 0\,Amps \tag{51}$$

Since $\dot{C}$ has units of conductance in mhos, $\mho$, its direction of change could either be POSITIVE or NEGATIVE. If $\dot{C}$ is negative, it has units of negative conductance or,

$$\dot{C} < 0\mho \tag{52}$$

Now, the instantaneous gravitationally induced power P_C of a fluctuating capacitor $\dot{C}$ is

$$P_C(t) = i_C(t)V_C = V_C^{\,2}\,\dot{C} \tag{53}$$

So, for certain values of $\dot{C}$, the total instantaneous power P_C can be NEGATIVE or,

$$P_C(t) < 0 Watts \tag{54}$$

Then, integrating P_C with respect to time when the total power is less than zero watts results in NEGATIVE energy of capacitor C or,

$$E_C(t) = \int P_C \, dt = V_C^{\,2} \int \dot{C} \, dt = V_C^{\,2} \, C(t) < 0 \, Joules \tag{55}$$

If $\dot{C}$ is positive, it has units of positive conductance or,

$$\dot{C} > 0 \mho \tag{56}$$

Now, the instantaneous gravitationally induced power P_C of a fluctuating capacitor $\dot{C}$ is,

$$P_C(t) = i_C(t) V_C = V_C^{\,2} \, \dot{C} \tag{57}$$

So, for certain values of $\dot{C}$, the total instantaneous power P_C can be POSITIVE or

$$P_C(t) > 0 \, Watts \tag{58}$$

Then, integrating P_C with respect to time results in excess POSITIVE energy of capacitor C or,

$$E_C(t) = \int P_C \, dt = V_C^{\,2} \int \dot{C} \, dt = V_C^{\,2} \, C(t) > 0 \, Joules \tag{59}$$

Equate this to the energy equivalent of mass E_M,

$$E_C(t) = E_M(t) \tag{60}$$

Then, the energy equivalent of mass (**gravitational energy**) E_C is,

$$E_C(t) = v_Y^{\,2} \, M_C(t) = V_C^{\,2} \, C(t) \tag{61}$$

By rearranging terms, the mass equivalent of energy M_C is,

$$M_C(t) = \frac{E_C(t)}{v_Y^{\,2}} = \frac{C(t) V_C^{\,2}}{v_Y^{\,2}} \tag{62}$$

$$v_Y^{\,2} = \frac{V_C^{\,2} \, C(t)}{M_C(t)} \tag{63}$$

The fluctuating mass equivalent of energy $\dot{M}_C$ is,

$$\dot{M}_C = \frac{\dot{E}_C}{v_Y^{\,2}} = \frac{V_C^{\,2} \, \dot{C}}{v_Y^{\,2}} \tag{64}$$

So, the gravitational capacitor model is,

$$f_C(t) = \dot{M}_C \, v_Y = \frac{\dot{E}_C}{v_Y^{\,2}} v_Y = \frac{\dot{E}_C}{v_Y} = \frac{V_C^{\,2} \, \dot{C}}{v_Y} \tag{65}$$

Letting $\dot{y} = dy/dt = v_Y$, the **Gravitational Capacitive Force Coupling Term** is,

$$f_C(t) = \frac{\dot{E}_C}{\dot{y}} \tag{66}$$

By rearranging terms, the fluctuating energy equivalent of mass $\dot{E}_C$ is,

$$\dot{E}_C = f_L(t)\,\dot{y} \tag{67}$$

The total energy E_C contained within matter is,

$$E_C(t) = M_C(t)\,c^2 \tag{68}$$

The change in this total energy $\dot{E}_C$ is,

$$\dot{E}_C = \dot{M}_C\,c^2 \tag{69}$$

By rearranging terms, the change in total mass $\dot{M}$ is,

$$\dot{M}_C = \frac{\dot{E}_C}{c^2} = \frac{f_C(t)\,\dot{y}}{c^2} = \frac{V_C^{\,2}\,\dot{C}}{\dot{y}^2} = \frac{V_C^{\,2}\,\dot{C}}{\frac{V_C^{\,2}\,C(t)}{M_C(t)}} = \frac{M_C(t)\,\dot{C}}{C(t)} \tag{70}$$

So, the time derivative form of the **Gravitational Capacitive Coupling Term** is,

$$\dot{C} = \frac{f_C(t)\,C(t)\,\dot{y}}{M_C(t)\,c^2} \tag{71}$$

The derivative form is,

$$dC = \frac{f_C\,C\,dy}{M_C\,c^2} \tag{72}$$

The difference form is,

$$\Delta C = \frac{f_C\,C\,\Delta y}{M_C\,c^2} \tag{73}$$

THE GRAVITATIONAL COUPLING OF A FLUCTUATING MASS, INDUCTOR OR CAPACITOR

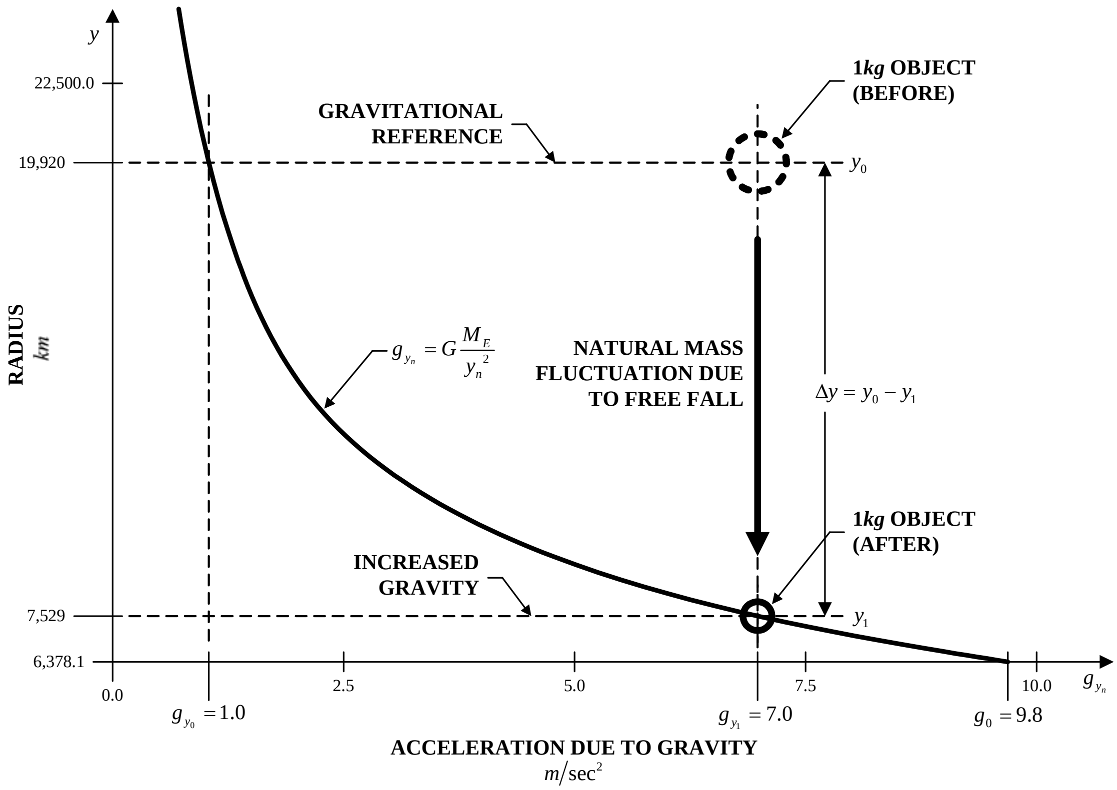


FIGURE 5. The natural mass fluctuation of an object due to gravitational free fall.

Natural universal mass attraction or classic Newtonian gravity is a force f_y that acts through a center of mass of the Earth with mass M_E and a test mass M separated by a distance y ,

$$f_y = \frac{G\,M_E\,M}{y^2} = g_y\,M \tag{74}$$

So, given,

Gravitational constant $G = 6.67259 \times 10^{-11}\,N\,m^2/kg^2$

Mass of the Earth $M_E = 5.9787 \times 10^{24}\,kg$

Radius of Earth $y_E = 6.3781 \times 10^6\,m$

The surface gravity g_0 of the Earth is,

$$g_0 = \frac{G\,M_E}{y_E^{\,2}} = \frac{\left(6.67259 \times 10^{-11}\,N\,m^2/kg^2\right)\left(5.9787 \times 10^{24}\,kg\right)}{\left(6.3781 \times 10^6\,m\right)^2} = 9.80665\,m/\sec^2 \tag{75}$$

The gravity g_{y_0} at GRAVITATIONAL REFERENCE position y_0 is,

$$g_{y_0} = \frac{G\,M_E}{{y_0}^2} = \frac{\left(6.67259\times10^{-11}\,N\,m^2/kg^2\right)\left(5.9787\times10^{24}\,kg\right)}{\left(19.920\times10^{6}\,m\right)^2} = 1.00537\,m/\sec^2 \tag{76}$$

The gravity g_{y_1} at position y_1 is,

$$g_{y_1} = \frac{G\,M_E}{{y_1}^2} = \frac{\left(6.67259\times10^{-11}\,N\,m^2/kg^2\right)\left(5.9787\times10^{24}\,kg\right)}{\left(7.529\times10^{6}\,m\right)^2} = 7.03767\,m/\sec^2 \tag{77}$$

Marmet (2001) equates force f_M produced by a fluctuating mass dM to the gravitational force f_y,

$$f_y(t) = f_M(t) \tag{78}$$

The derivative form of a fluctuating mass dM is,

$$dM = \frac{f_M\,dy}{c^2} = \frac{g_y\,M\,dy}{c^2} \tag{79}$$

The derivative form of the gravitational RED SHIFT (or BLUE SHIFT) of mass dM/M and the energy equivalent of mass dE/E displaced dy within a given gravity well g_y is,

$$\frac{dM}{M} = \frac{dE_M}{E_M} = \frac{g_y\,dy}{c^2} \tag{80}$$

Marmet states the equation above shows a "calculated change of energy levels as a function of gravitational potential is in perfect agreement with the Pound and Rebka and also the Pound and Snider experiments". Therefore, energy increases as a function of downward or positive displacement within a given gravity well.

Let $dY_{g_y} = g_y\,dy$, the exponential solutions of the derivative form of mass and energy equivalent of mass are,

$$\int_{M_{y_0}}^{M_{y_1}} \frac{1}{M}\,dM = \frac{1}{c^2}\int_{g_{y_0}\,y_0}^{g_{y_1}\,y_1} dY_{g_y} \tag{81}$$

$$\ln\left(M\right)\Big|_{M_{y_0}}^{M_{y_1}} = \frac{1}{c^2}\,Y_{g_y}\Big|_{g_{y0}\,y_0}^{g_{y1}\,y_1} \tag{82}$$

$$\ln\left(M_{y_1}\right) - \ln\left(M_{y_0}\right) = \ln\left(\frac{M_{y_1}}{M_{y_0}}\right) = \frac{1}{c^2}\left(g_{y_1}\,y_1 - g_{y_0}\,y_0\right) \tag{83}$$

$$\frac{M_{y_1}}{M_{y_0}} = e^{\left(\frac{g_{y_1}\,y_1 - g_{y_0}\,y_0}{c^2}\right)} \tag{84}$$

$$M_{y_1} = M_{y_0}\,e^{\left(\frac{g_{y_1}\,y_1 - g_{y_0}\,y_0}{c^2}\right)} \tag{85}$$

$$E_{M\,y_1} = E_{M\,y_0} e^{\left(\frac{g_{y_1} y_1 - g_{y_0} y_0}{c^2}\right)} \tag{86}$$

So, the Pound, Rebka and Snider experiment used Mossbaurer spectroscopy to measure the electromagnetic gravitational RED SHIFT (or BLUE SHIFT) of $14.4\,keV$ gamma rays emitted from Fe^{57} through a vertical distance of $22.6\,m$. With the gamma rays emitted upward, they showed the RED SHIFT was within one percent (1%) of this result,

$$SHIFT = \frac{\Delta M}{M} = \frac{\Delta E_M}{E_M} = \frac{g_{y_1}\,y_1 - g_{y_0}\,y_0}{c^2} = \frac{G\,M_E\left(\frac{1}{y_1} - \frac{1}{y_0}\right)}{c^2} \tag{87}$$

$$SHIFT = \frac{\left(6.67259\times10^{-11}\,N\,m^2/kg^2\right)\left(5.9787\times10^{24}\,kg\right)\left(\frac{1}{\left(6.3781226\times10^6\,m\right)} - \frac{1}{\left(6.3781000\times10^6\,m\right)}\right)}{(2.99792458\times10^8\,m/\sec)^2} \tag{88}$$

$$SHIFT = -2.465961\times10^{-15} \tag{89}$$

With the gamma rays emitted downward, they showed the BLUE SHIFT was,

$$SHIFT = \frac{\left(6.67259\times10^{-11}\,N\,m^2/kg^2\right)\left(5.9787\times10^{24}\,kg\right)\left(\frac{1}{\left(6.3781000\times10^6\,m\right)} - \frac{1}{\left(6.3781226\times10^6\,m\right)}\right)}{(2.99792458\times10^8\,m/\sec)^2} \tag{90}$$

$$SHIFT = 2.465961\times10^{-15} \tag{91}$$

Now, the force f_L produced by a fluctuating inductor dL is equated to the gravitational force f_y,

$$f_y(t) = f_L(t) \tag{92}$$

The derivative form of a fluctuating inductor dL is,

$$dL = \frac{f_L\,L\,dy}{M_L\,c^2} = \frac{g_y\,L\,dy}{c^2} \tag{93}$$

The derivative form of the gravitational RED SHIFT (or BLUE SHIFT) of inductor dL/L displaced dy within a given gravity well g_y is,

$$\frac{dL}{L} = \frac{g_y\,dy}{c^2} \tag{94}$$

Let $dY_{g_y} = g_y\,dy$, the exponential solution of the derivative form of an inductor is,

$$\int_{L_{y_0}}^{L_{y_1}} \frac{1}{L}\,dL = \frac{1}{c^2}\int_{g_{y_0}\,y_0}^{g_{y_1}\,y_1} dY_{g_y} \tag{95}$$

$$\ln(L)\Big|_{L_{y_0}}^{L_{y_1}} = \frac{1}{c^2} Y_{g_y}\Big|_{g_{y0}\,y_0}^{g_{y1}\,y_1} \tag{96}$$

$$\ln\left(L_{y_1}\right) - \ln\left(L_{y_0}\right) = \ln\left(\frac{L_{y_1}}{L_{y_0}}\right) = \frac{1}{c^2}\left(g_{y_1}\; y_1 - g_{y_0}\; y_0\right) \tag{97}$$

$$\frac{L_{y_1}}{L_{y_0}} = e^{\left(\frac{g_{y_1}\,y_1 - g_{y_0}\,y_0}{c^2}\right)} \tag{98}$$

$$L_{y_1} = L_{y_0}\, e^{\left(\frac{g_{y_1}\,y_1 - g_{y_0}\,y_0}{c^2}\right)} \tag{99}$$

So, the gravitational RED SHIFT (or BLUE SHIFT) of an inductor L is,

$$SHIFT = \frac{\Delta L}{L} = \frac{g_{y_1}\; y_1 - g_{y_0}\; y_0}{c^2} = \frac{G\,M_E\left(\frac{1}{y_1} - \frac{1}{y_0}\right)}{c^2} \tag{100}$$

Now, the force f_C produced by a fluctuating capacitor dC is equated to the gravitational force f_y,

$$f_y(t) = f_C(t) \tag{101}$$

The derivative form of a fluctuating inductor dC is,

$$dC = \frac{f_C\; C\, dy}{M_C\; c^2} = \frac{g_y\; C\, dy}{c^2} \tag{102}$$

The derivative form of the gravitational RED SHIFT (or BLUE SHIFT) of capacitor dC/C displaced dy within a given gravity well g_y is,

$$\frac{dC}{C} = \frac{dE_C}{E_C} = \frac{g_y\; dy}{c^2} \tag{103}$$

Let $dY_{g_y} = g_y\; dy$, the exponential solution of the derivative form of a capacitor is,

$$\int_{C_{y_0}}^{C_{y_1}} \frac{1}{C}\, dC = \frac{1}{c^2}\int_{g_{y_0}\,y_0}^{g_{y_1}\,y_1} dY_{g_y} \tag{104}$$

$$\ln(C)\Big|_{C_{y_0}}^{C_{y_1}} = \frac{1}{c^2} Y_{g_y}\Big|_{g_{y0}\,y_0}^{g_{y1}\,y_1} \tag{105}$$

$$\ln\left(C_{y_1}\right) - \ln\left(C_{y_0}\right) = \ln\left(\frac{C_{y_1}}{C_{y_0}}\right) = \frac{1}{c^2}\left(g_{y_1}\; y_1 - g_{y_0}\; y_0\right) \tag{106}$$

$$\frac{C_{y_1}}{C_{y_0}} = e^{\left(\frac{g_{y_1} y_1 - g_{y_0} y_0}{c^2}\right)} \tag{107}$$

$$C_{y_1} = C_{y_0} e^{\left(\frac{g_{y_1} y_1 - g_{y_0} y_0}{c^2}\right)} \tag{108}$$

So, the gravitational RED SHIFT (or BLUE SHIFT) of a capacitor C is,

$$SHIFT = \frac{\Delta C}{C} = \frac{g_{y_1} \; y_1 - g_{y_0} \; y_0}{c^2} = \frac{G M_E \left(\frac{1}{y_1} - \frac{1}{y_0}\right)}{c^2} \tag{109}$$

NATURAL RELATIVITY THEORY

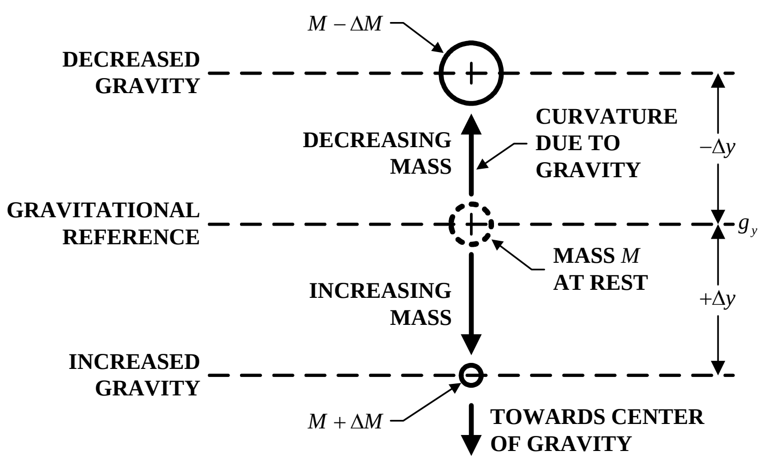


FIGURE 6. A change of *relativistic* mass due to gravity.

The establishment of a GRAVITATIONAL REFERENCE is defined as a fixed point of reference within a given gravity well g_y. This point may be located in a plane of equipotential surface of gravity, and is used throughout this paper. *Natural relativistic* changes of mass, volume, frequency, energy, etc. fluctuate or curve as a function of displacement $\pm\Delta y$ from this point of reference. This displacement defines a new point within a new plane of equipotential surface of gravity $g_{\pm\Delta y}$, and the gravity at that point may be increased or decreased based upon the sign of the displacement. Marmet (2001) invokes the principle of mass-energy conservation regarding the displacement of matter between planes. For example, an object of mass M displaced a distance Δy changes back to its original mass when returned to its original position within a given gravity well. Therefore, a new and simplified relativity model is introduced. For **gravitational energy** systems, and given an equipotential surface of gravity reference, the following parameters including *relativistic* mass $\pm\Delta M$, *relativistic* inductance $\pm\Delta L$, and *relativistic* capacitance $\pm\Delta C$, fluctuate or curve between equipotential surfaces of gravity by displacement $\pm\Delta y$. Again, the kinetic energy of gravitational **energy** systems is assumed to be zero. So, given a common equipotential surface of gravity reference g_y, an increase in gravity causes a *natural relativistic* increase in mass, energy equivalent of mass (**gravitational energy**), inductance, and capacitance. Likewise, a decrease in gravity causes a *natural relativistic* decrease in the same metrics.

Given an object with a rest mass M_{y_0} , an equivalent energy of the rest mass $E_{M\,y_0}$, an inductance L_{y_0} , and a capacitance C_{y_0} , the new rest mass M_{y_1} , the new equivalent energy of the rest mass E_{My_0} , the new inductance L_{y_1} , and the new capacitance C_{y_1} are,

$$M_{y_1} = \gamma_{NR} M_{y_0} = M_{y_0} \pm \Delta M_{y_0} \tag{110}$$

$$E_{My_1} = \gamma_{NR} E_{My_0} = E_{My_0} \pm \Delta E_{My0} \tag{111}$$

$$L_{y_1} = \gamma_{NR} L_{y_0} = L_{y_0} \pm \Delta L_{y_0} \tag{112}$$

$$C_{y_1} = \gamma_{NR} C_{y_0} = C_{y_0} \pm \Delta C_{y_0} \tag{113}$$

The *natural relativistic* gamma γ_{NR} is,

$$\gamma_{NR} = e^{\left(\frac{g_{y_1}\, y_1 - g_{y_0}\, y_0}{c^2}\right)} \tag{114}$$

The difference forms and the exponential forms of the *natural relativistic* mass M_{y_1} model, energy equivalent of mass E_{My_1} model, inductor L_{y_1} model, and capacitor C_{y_1} model at position $\pm\Delta y$ within a given gravity well g_y are,

$$M_{y_1} = M_{y_0} \pm \Delta M_{y_0} = M_{y_0}\left(1 \pm \frac{g_{y_1}\, y_1 - g_{y_0}\, y_0}{c^2}\right) = M_{y_0}\, e^{\left(\frac{g_{y_1}\, y_1 - g_{y_0}\, y_0}{c^2}\right)} \tag{115}$$

$$E_{My_1} = E_{My_0} + \Delta E_{My_0} = E_{My_0}\left(1 \pm \frac{g_{y_1}\, y_1 - g_{y_0}\, y_0}{c^2}\right) = E_{My_0}\, e^{\left(\frac{g_{y_1}\, y_1 - g_{y_0}\, y_0}{c^2}\right)} \tag{116}$$

$$L_{y_1} = L_{y_0} + \Delta L_{y_0} = L_{y_0}\left(1 \pm \frac{g_{y_1}\, y_1 - g_{y_0}\, y_0}{c^2}\right) = L_{y_0}\, e^{\left(\frac{g_{y_1}\, y_1 - g_{y_0}\, y_0}{c^2}\right)} \tag{117}$$

$$C_{y_1} = C_{y_0} \pm \Delta C_{y_0} = C_{y_0}\left(1 \pm \frac{g_{y_1}\, y_1 - g_{y_0}\, y_0}{c^2}\right) = C_{y_0}\, e^{\left(\frac{g_{y_1}\, y_1 - g_{y_0}\, y_0}{c^2}\right)} \tag{118}$$

In summary, Natural Relativity (NR) Theory is the **primary gravitational effect**.

SPECIAL RELATIVITY THEORY

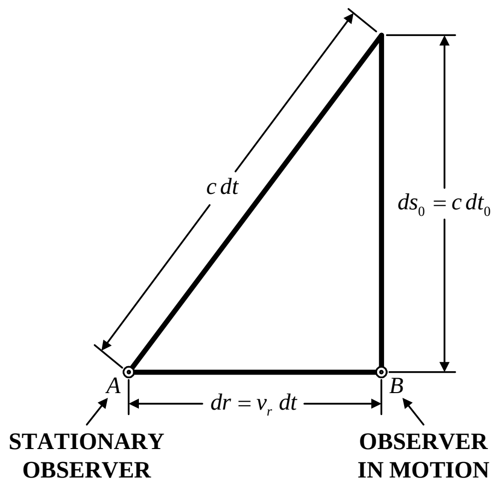


FIGURE 7. The definition of a space-time interval relative to Observer B in motion.

Einstein (1905) formulated his theory of *special relativity* and is described as Observer B, moving at velocity v_r relative to a stationary Observer A, undergoes a relativistic effect. This effect changes the mass, lengths, time intervals, frequencies, and energy of the observer in motion. It's expressed as a Pythagorean-type quantity called a space-time interval, and evaluates as a Lorentz temporal correction shown below.

$$ds_0{}^2 + dr^2 = c^2 dt^2 \tag{119}$$

$$ds_0{}^2 = c^2 dt^2 - dr^2 \tag{120}$$

Since the velocity v_r of Observer B is,

$$v_r = \dot{r} = \frac{dr}{dt} \tag{121}$$

$$dr = v_r \, dt \tag{122}$$

So,

$$ds_0{}^2 = c^2 dt^2 - v_r{}^2 dt^2 \tag{123}$$

$$ds_0 = \sqrt{c^2 dt^2 - \frac{c^2 dt^2 v_r{}^2}{c^2}} = c\,dt\sqrt{1 - \frac{v_r{}^2}{c^2}} \tag{124}$$

Since the speed of light c is,

$$c = \dot{s}_0 = \frac{ds_0}{dt} \tag{125}$$

$$ds_0 = c\,dt_0 \tag{126}$$

So, the Lorentz temporal correction relative to Observer B is,

$$c\,dt_0 = c\,dt\sqrt{1-\frac{{v_r}^2}{c^2}} \tag{127}$$

$$dt_0 = dt\sqrt{1-\frac{{v_r}^2}{c^2}} \tag{128}$$

$$dt = \frac{dt_0}{\sqrt{1-\frac{{v_r}^2}{c^2}}} \tag{129}$$

Therefore, relative to Observer B in motion with a time interval dt_{B_0}, stationary Observer A's clock with a time interval dt_A will be ticking faster or BLUE SHIFTED, and have the following Lorentz temporal correction,

$$dt_A = \frac{dt_{B_0}}{\sqrt{1-\frac{{v_r}^2}{c^2}}} \tag{130}$$

$$dt_A > dt_{B_0} \tag{131}$$

Likewise, relative to stationary Observer A with a time interval dt_{A_0}, Observer B's clock at time interval dt_B will be ticking slower or RED SHIFTED, and have the following Lorentz temporal correction,

$$dt_B = dt_{A_0}\sqrt{1-\frac{{v_r}^2}{c^2}} \tag{132}$$

$$dt_B < dt_{A_0} \tag{133}$$

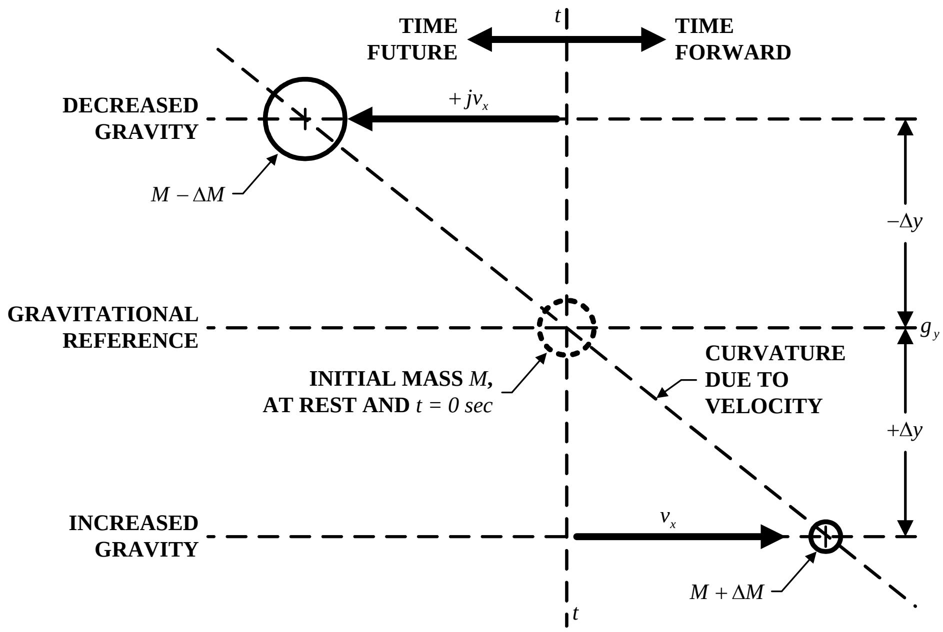


FIGURE 8. A change of *relativistic* mass due to velocity.

Given the rest mass of an object M , the *special relativistic* mass M_v model presented by Einstein (1905) shows an object moving at velocity v_x is,

$$M_v = \frac{M}{\sqrt{1-\frac{{v_x}^2}{c^2}}} = \gamma M = M + dM \tag{134}$$

The gamma γ is,

$$\gamma = \frac{1}{\sqrt{1-\frac{{v_x}^2}{c^2}}} \tag{135}$$

Implementing the binomial expansion of the above equation,

$$\gamma_{SR} = 1 + \frac{1}{2}\frac{{v_x}^2}{c^2} + \frac{3}{8}\frac{{v_x}^4}{c^4} + \frac{5}{16}\frac{{v_x}^6}{c^6} + \frac{35}{128}\frac{{v_x}^8}{c^8} + ... \tag{136}$$

Using the 1st order term of the expansion shown above where $v_x \ll c$, the new *special relativistic* gamma γ_{SR} is,

$$\gamma_{SR} = 1 + \frac{{v_x}^2}{2c^2} \tag{137}$$

The derivative form of the *special relativistic* mass M_v model moving at velocity v_x is,

$$M_v = \gamma_{SR} M = M \pm dM = M\left(1 \pm \frac{{v_x}^2}{2c^2}\right) \tag{138}$$

The derivative form of a fluctuating mass dM is,

$$dM = \frac{M\,{v_x}^2}{2\,c^2} \tag{139}$$

An object can move at a *real* (i.e., time-forward) velocity v_x, at an *imaginary* (i.e., time-future) velocity $j\,v_x$, or at a velocity that is a combination of the two. The *real* and *imaginary* components are rotated about the temporal axis and therefore, can be described as *complex* motion. The rotation is given as $0° \le \theta \le 90°$, where the *real* axis is $\theta = 0°$ and the *imaginary* time-future axis is $\theta = 90°$. The *complex* number uses the Euler's identity $e^{j\theta}$, which functions as a temporal rotation operator.

$$v_x = v\,e^{j\theta} = v\cos\theta + j\,v\sin\theta \tag{140}$$

So, the derivative form of the inertial RED SHIFT (or BLUE SHIFT) of mass dM/M, energy equivalent of mass dE_M/E_M, inductor dL/L, or capacitor dC/C of an object moving at a *real* velocity v_x or a *complex* velocity $j\,v_x$ is,

$$SHIFT = \frac{dM}{M} = \frac{dE_M}{E_M} = \frac{dL}{L} = \frac{dC}{C} = \frac{{v_x}^2}{2\,c^2} \tag{141}$$

The exponential solution of the derivative form of mass is,

$$\int_{M_0}^{M_v} \frac{1}{M}\,dM = \frac{{v_x}^2}{2\,c^2} \tag{142}$$

$$\ln\left(M\right)\Big|_{M_0}^{M_v} = \frac{{v_x}^2}{2\,c^2} \tag{143}$$

$$\ln\left(M_v\right) - \ln\left(M_0\right) = \ln\left(\frac{M_v}{M_0}\right) = \frac{{v_x}^2}{2\,c^2} \tag{144}$$

$$\frac{M_v}{M_0} = e^{\left(\frac{{v_x}^2}{2c^2}\right)} \tag{145}$$

$$M_v = M_0\,e^{\left(\frac{{v_x}^2}{2c^2}\right)} \tag{146}$$

$$E_{Mv} = E_{M0}\,e^{\left(\frac{{v_x}^2}{2c^2}\right)} \tag{147}$$

$$L_v = L_0\,e^{\left(\frac{{v_x}^2}{2c^2}\right)} \tag{148}$$

$$C_v = C_0\,e^{\left(\frac{{v_x}^2}{2c^2}\right)} \tag{149}$$

So, the inertial RED SHIFT (or BLUE SHIFT) of a mass M , an energy equivalent of mass E_M , an inductor L , and a capacitor C is,

$$SHIFT = \frac{\Delta M}{M} = \frac{\Delta E_M}{E_M} = \frac{\Delta L}{L} = \frac{\Delta C}{C} = \frac{{v_x}^2}{2\,c^2} \tag{150}$$

The *special relativistic* gamma γ_{SR} is,

$$\gamma_{SR} = e^{\left(\frac{{v_x}^2}{2\,c^2}\right)} \tag{151}$$

The difference forms and the exponential forms of the *special relativistic* mass M_v model, energy equivalent of mass E_{Mv} model, inductor L_v model, and capacitor C_v model of an object moving at a *real* velocity v_x or a *complex* velocity $j\,v_x$ are,

$$M_v = M_0 \pm \Delta M_0 = M_0\left(1 \pm \frac{{v_x}^2}{2\,c^2}\right) = M_0\, e^{\left(\frac{{v_x}^2}{2\,c^2}\right)} \tag{152}$$

$$E_{M\,v} = E_{M\,0} \pm \Delta E_{M\,0} = E_{M\,0}\left(1 \pm \frac{{v_x}^2}{2\,c^2}\right) = E_{M\,0}\, e^{\left(\frac{{v_x}^2}{2\,c^2}\right)} \tag{153}$$

$$L_v = L_0 \pm \Delta L_0 = L_0\left(1 \pm \frac{{v_x}^2}{2\,c^2}\right) = L_0\, e^{\left(\frac{{v_x}^2}{2\,c^2}\right)} \tag{154}$$

$$C_v = C_0 + \Delta C_0 = C_0\left(1 \pm \frac{{v_x}^2}{2\,c^2}\right) = C_0\, e^{\left(\frac{{v_x}^2}{2\,c^2}\right)} \tag{155}$$

DID EINSTEIN GET IT RIGHT, OR NOT?

Einstein's General Relativity Theory (1916) equates Newton's second law of motion, $f = m_i a$, where m_i is the inertial mass to Newton's gravitational force, $f = m_g g$, where m_g is the gravitational mass. Is this concept correct? Einstein used this to formulate his **equivalence principle** and stated, "There is no experiment a person could conduct in a small volume of space that would distinguish between a gravitational field and an equivalent uniform acceleration". Is this statement correct? Lets test Einstein's principle in the following thought experiment:

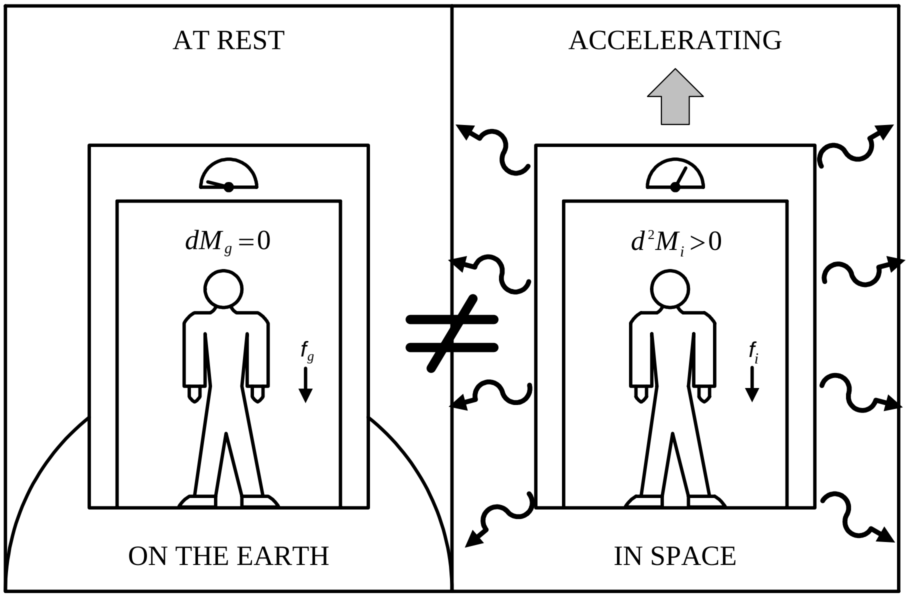


FIGURE 9. An elevator at rest on the Earth is NOT equivalent to an elevator accelerating in space.

As shown above, the *natural relativistic* mass fluctuation of an elevator at rest on the surface of the Earth is zero, or $dM = 0$. However, the second order *special relativistic* mass fluctuation of the same elevator accelerating in space is non-zero or $d^2M > 0$, and as a consequence, radiates electromagnetic waves. According to Woodward (1998), radiation reaction is observed in bodies being accelerated based upon Newton's second law of motion, $f_0 = m a$. Therefore, given this scenario, the gravitational mass can't be equivalent to its' inertial mass due to their differences in mass fluctuations.

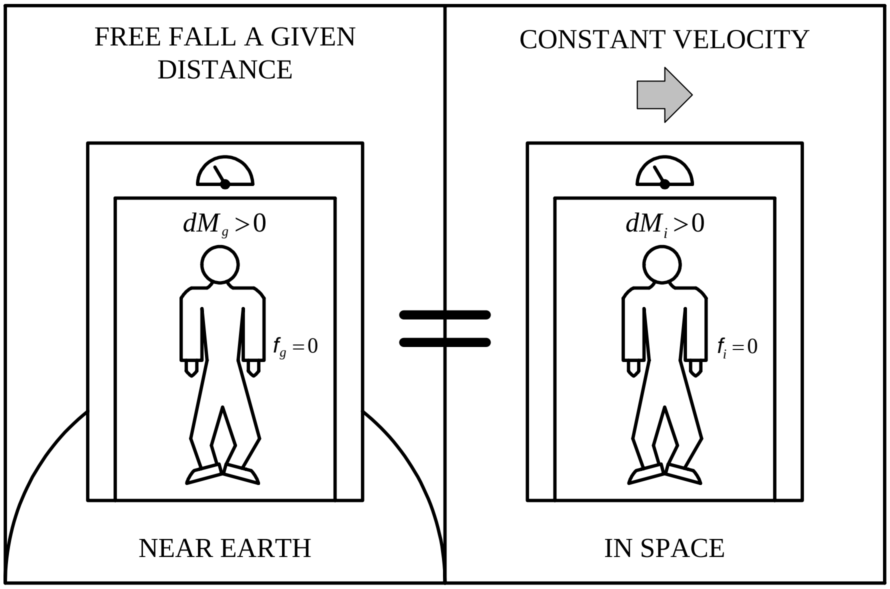


FIGURE 10. An elevator in free fall above the Earth is equivalent to an elevator moving at constant velocity in space.

As shown above, an elevator traveling a distance $+dy$ in free fall and the same elevator moving at a constant velocity v_x at right angles to free fall produce virtually no radiation reaction. So, given this second scenario, these mass fluctuations are considered equivalent, hence, establishing a new **Principle of Equivalence Theorem**.

A NEW PRINCIPLE OF EQUIVALENCE THEOREM

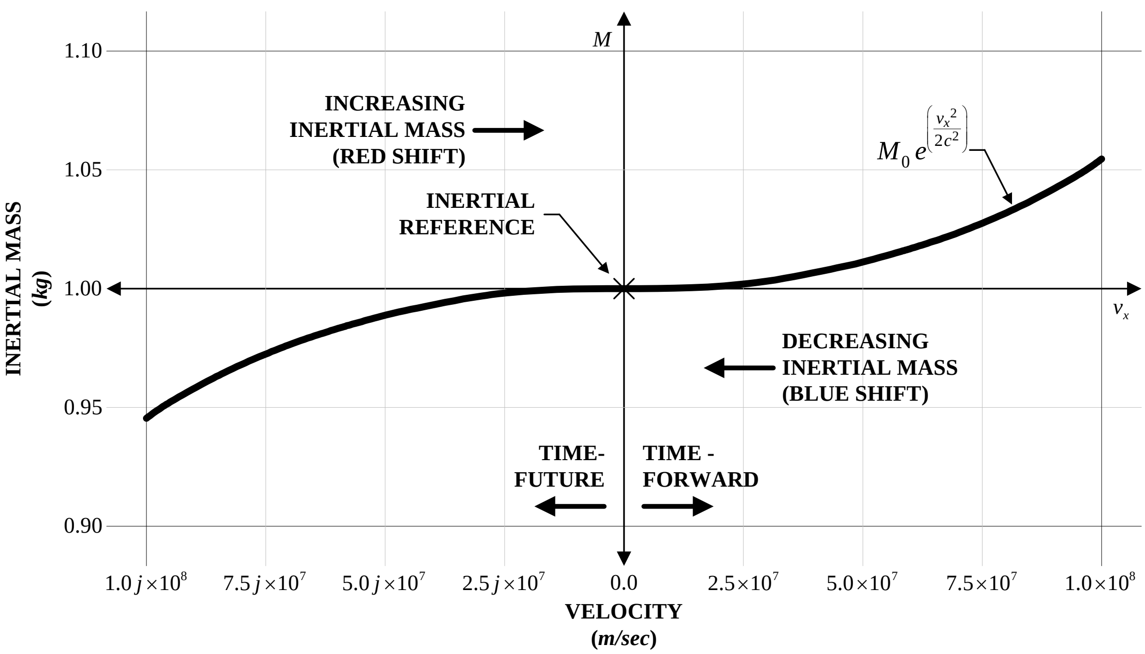


FIGURE 11. Velocity profile of a 1*kg* inertial mass.

The inertial RED SHIFT of an object due to velocity v_x is,

$$SHIFT = \frac{\Delta M}{M} = \frac{{v_x}^2}{2c^2} \tag{156}$$

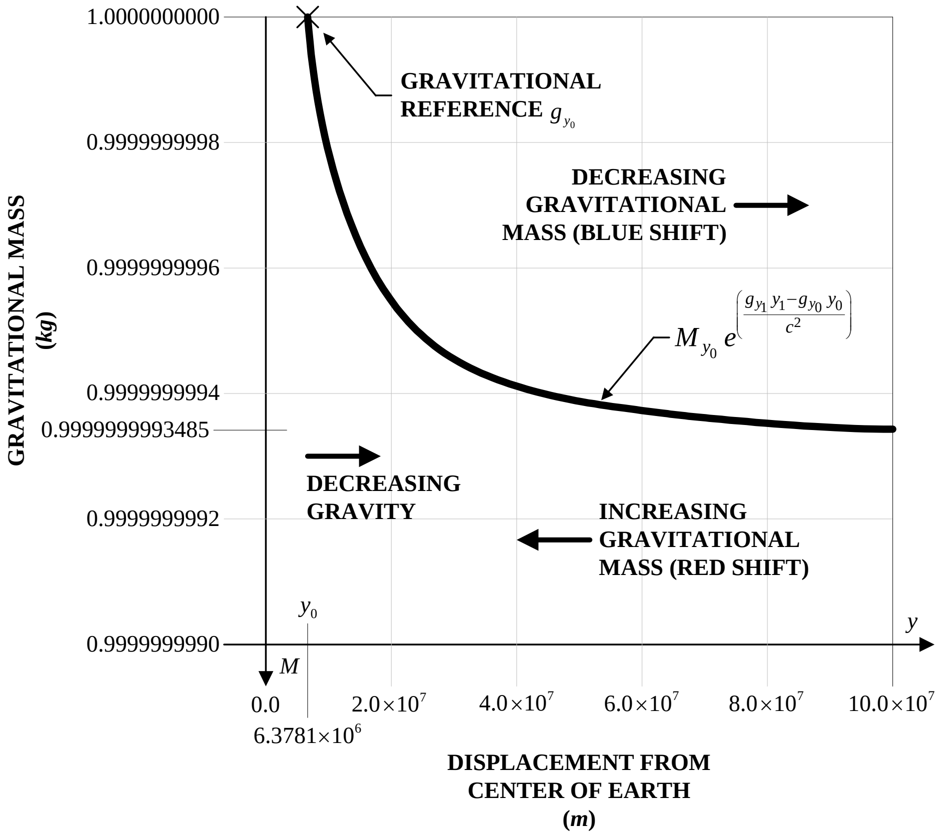


FIGURE 12. Displacement profile of a 1*kg* gravitational mass due to Earth's gravity well.

And since the gravitational RED SHIFT of the same object due to gravity g_y is,

$$SHIFT = \frac{\Delta M}{M} = \frac{g_{y_1}\, y_1 - g_{y_0}\, y_0}{c^2} = \frac{G M_E \left(\frac{1}{y_1} - \frac{1}{y_0}\right)}{c^2} \tag{157}$$

By equating an inertial RED SHIFT to a gravitational RED SHIFT, a new **Principle of Equivalence Theorem** is determined as,

$$\frac{{v_x}^2}{2c^2} = \frac{g_{y_1}\, y_1 - g_{y_0}\, y_0}{c^2} = \frac{G M_E \left(\frac{1}{y_1} - \frac{1}{y_0}\right)}{c^2} \tag{158}$$

So, an object displaced $\pm\Delta y$ within the Earth's gravity well g_y is equivalent to the same object moving at *complex* velocity v_x,

$$v_x = \sqrt{2\left(g_{y_1}\, y_1 - g_{y_0}\, y_0\right)} = \sqrt{2\,G\,M_E\left(\frac{1}{y_1} - \frac{1}{y_0}\right)} \qquad (159)$$

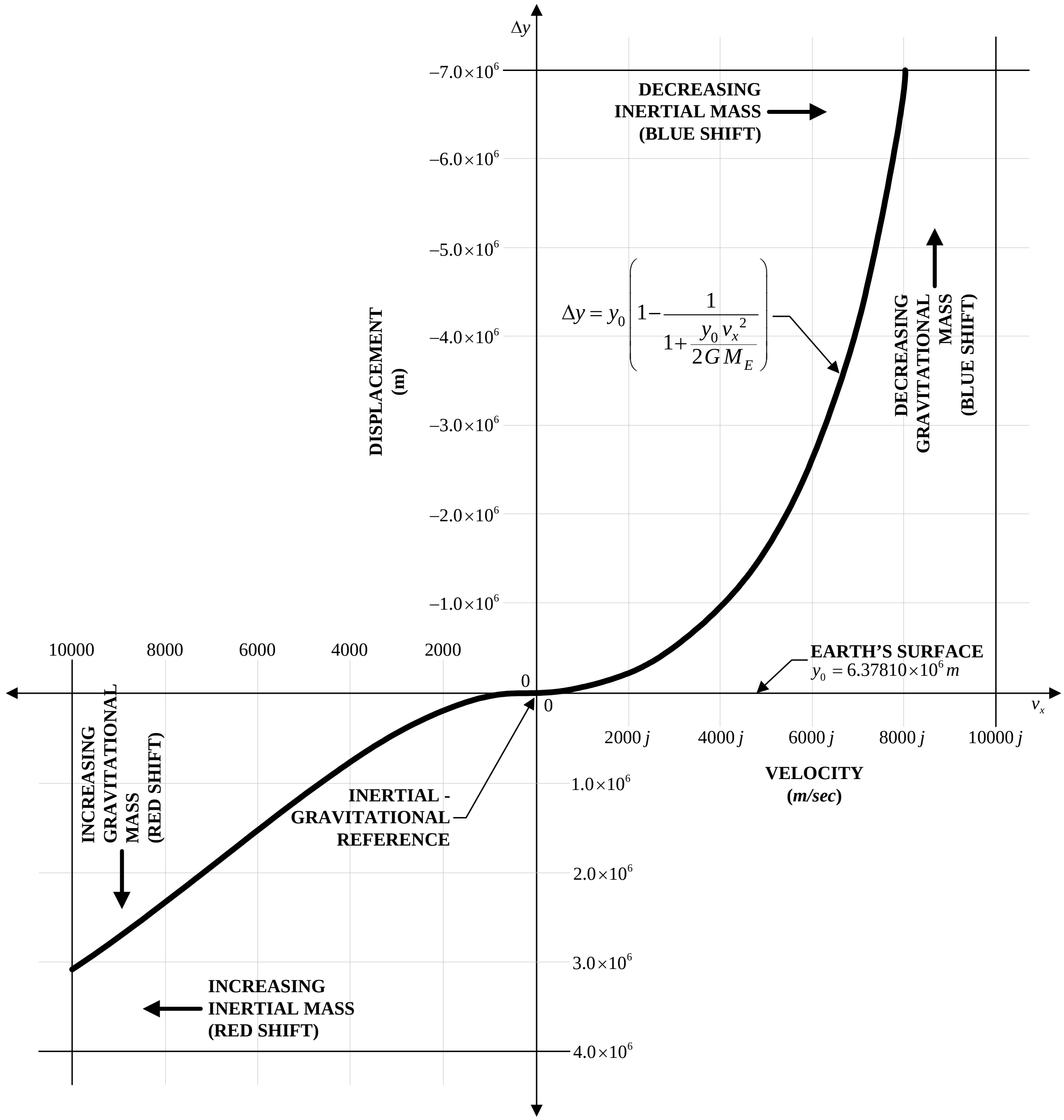


FIGURE 13. Equating an inertial RED (BLUE) SHIFT to a gravitational RED (BLUE) SHIFT.

As shown above, if the displacement $\Delta y = y_0 - y_1$ of an object is POSITIVE, then the object is moving at a *real* velocity v_x. However, if the displacement Δy is NEGATIVE, then the same object is moving at a *complex* (i.e., time-future) velocity $\sqrt{-1}\,v_x$, or velocity $j\,v_x$, where $j = \sqrt{-1}$. The *real* and *imaginary* components are rotated about the temporal axis as a *complex* velocity. The rotation is given as $0° \le \theta \le 90°$, where the *real* axis is $\theta = 0°$ and the *imaginary* time-future axis is $\theta = 90°$. The *complex* number uses the Euler's identity $e^{j\theta}$, which functions as a temporal rotation operator.

$$v_x = v\,e^{j\theta} = v\cos\theta + j\,v\sin\theta \tag{160}$$

So, given an object moving at a *complex* velocity v_x, the equivalent displacement to position y_1 within the Earth's gravity well g_y, where $0 < y_1 \le \infty$ or $-1 \le \frac{y_0\,v_x^{\,2}}{2\,G\,M_E}$ is,

$$y_1 = \frac{1}{g_{y_1}}\left(g_{y_0}\;y_0 + \frac{v_x^{\,2}}{2}\right) = \frac{y_0}{1+\frac{y_0\,v_x^{\,2}}{2\,G\,M_E}} \tag{161}$$

Therefore, the equivalent displacement Δy within the Earth's gravity well g_y is,

$$\Delta y = y_0 - y_1 = y_0 - \frac{y_0}{1+\frac{y_0\,v_x^{\,2}}{2\,G\,M_E}} = y_0\left(1-\frac{1}{1+\frac{y_0\,v_x^{\,2}}{2\,G\,M_E}}\right) \tag{162}$$

The equivalent maximum *complex* velocity $v_{x\max}$ at $y_1 = \infty$ is

$$1+\frac{y_0\,v_x^{\,2}}{2\,G\,M_E} = 0 \tag{163}$$

$$v_{x\max} = \sqrt{-\frac{2\,G\,M_E}{y_0}} \tag{164}$$

So, given the equivalent maximum *complex* velocity $v_{x\max}$, the minimum mass $M_{\min}$ at $y_1 = \infty$ is,

$$M_{\min} = M_{y_0}\left(1+\frac{v_x^{\,2}}{2\,c^2}\right) = M_{y_0}\left(1-\frac{G\,M_E}{y_0\,c^2}\right) = M_{y_0}\;e^{\left(-\frac{G\,M_E}{y_0\,c^2}\right)} \tag{165}$$

In summary, this new **Principle of Equivalence Theorem** describes an object moving at one-half the square of a *real* velocity v_x is equivalent to the same object having fallen *down* a displacement $+dy$ within a given gravity well g_y. This object *naturally* acquires more *relativistic* mass, inductance and capacitance as it moves at a *real* velocity v_x, and augments its' own gravitation with other objects. On the other hand, the same object moving at one-half the square of a *complex* velocity $j\,v_x$ is equivalent to the same object having fallen *up* a displacement $-dy$ in the same gravity well. This object *naturally* loses more *relativistic* mass, inductance and capacitance as it moves at a *complex* velocity $j\,v_x$, and diminishes its' own gravitation with other objects. Therefore, *special relativity* is considered to be a **secondary gravitational effect**.

Example 1. Given the velocity profile above of an object having a mass M_0 moving at a *special relativistic* time-forward velocity v_x, compute the new *special relativistic* inertial mass M_v.

So, given,
Direction of time $\theta = 0°$
Mass of object $M_0 = 1.0\,kg$
Velocity of object $v = 1.0 \times 10^8\ m/\sec$

The time-forward velocity v_x of an object is,

$$v_x = v\,e^{j\theta} = \left(1.0 \times 10^8\ m/\sec\right) e^{j0°} = 1.0 \times 10^8\ m/\sec \tag{166}$$

The new *special relativistic* inertial mass M_v is,

$$M_v = M_0\,e^{\left(\frac{v_x^2}{2c^2}\right)} = \left(1.0\,kg\right) e^{\left(\frac{\left(1.0\times10^8\ m/\sec\right)^2}{2\left(2.99793\times10^8\ m/\sec\right)^2}\right)} \tag{167}$$

$$M_v = 1.05721\,kg \tag{168}$$

The inertial mass of the object was increased by,

$$M_v - M_0 = \left(1.05721\,kg\right) - \left(1.0\,kg\right) = 0.05721\,kg \tag{169}$$

Example 2. Given the velocity profile above of an object having a mass M_0 moving at a *special relativistic* velocity time-future $j\,v_x$, compute the new *special relativistic* inertial mass M_v.

So, given,
Direction of time $\theta = 90°$
Mass of object $M_0 = 1.0\,kg$
Velocity of object $v = 1.0 \times 10^8\ m/\sec$

The time-future velocity v_x of an object is,

$$v_x = v\,e^{j\theta} = \left(1.0 \times 10^8\ m/\sec\right) e^{j90°} = 1.0\,j \times 10^8\ m/\sec \tag{170}$$

The new *special relativistic* inertial mass M_v is,

$$M_v = M_0\,e^{\left(\frac{v_x^2}{2c^2}\right)} = \left(1.0\,kg\right) e^{\left(\frac{\left(1.0\,j\times10^8\ m/\sec\right)^2}{2\left(2.99793\times10^8\ m/\sec\right)^2}\right)} \tag{171}$$

$$M_v = 0.94589\,kg \tag{172}$$

The inertial mass of the object was reduce by,

$$M_v - M_0 = \left(0.94589\,kg\right) - \left(1.0\,kg\right) = -0.05411\,kg \tag{173}$$

Example 3. Given the gravitational profile above of an object having a mass M_{y_0} displaced to a position $-\Delta y$ within Earth's gravity well g_y, compute the new *natural relativistic* gravitational mass M_{y_1}.

So, given,

Mass of object $M_{y_0} = 1.0\,kg$

Object on Earth's surface $y_0 = 6.3781 \times 10^6\,m$

Object displaced to $y_1 = 1.0 \times 10^8\,m$

Speed of light $c = 2.99792458 \times 10^8\,m/\sec$

Gravitational constant $G = 6.67260 \times 10^{-11}\,N\,m^2/kg^2$

Mass of the Earth $M_E = 5.9787 \times 10^{24}\,kg$

The acceleration due to gravity at surface of Earth is,

$$g_{y_0} = \frac{G\,M_E}{{y_0}^2} = \frac{\left(6.67260 \times 10^{-11}\,N\,m^2/kg^2\right)\left(5.9787 \times 10^{24}\,kg\right)}{\left(6.3781 \times 10^6\,m\right)^2} = 9.80665\,m/\sec^2 \tag{174}$$

The acceleration due to gravity at altitude $y_1 = 1.0 \times 10^8\,m$ above the Earth is,

$$g_{y_1} = \frac{G\,M_E}{{y_1}^2} = \frac{\left(6.67260 \times 10^{-11}\,N\,m^2/kg^2\right)\left(5.9787 \times 10^{24}\,kg\right)}{\left(1.0 \times 10^8\,m\right)^2} = 0.039894\,m/\sec^2 \tag{175}$$

Given the exponential solution of the *natural relativistic* mass model, the new gravitational mass is,

$$M_{y_1} = M_{y_0}\,e^{\left(\frac{g_{y_1}\,y_1 - g_{y_0}\,y_0}{c^2}\right)} = \left(1.0\,kg\right)e^{\left(\frac{\left(0.039894\,m/\sec^2\right)\left(1.0\times 10^8\,m\right) - \left(9.80665\,m/\sec^2\right)\left(6.3781\times 10^6\,m\right)}{\left(2.99792458\times 10^8\,m/\sec\right)^2}\right)} \tag{176}$$

$$M_{y_1} = 0.9999999993485\,kg \tag{177}$$

The gravitational mass of the object was reduced by,

$$M_{y_1} - M_{y_0} = \left(0.9999999993485\,kg\right) - \left(1.0\,kg\right) = -6.516 \times 10^{-10}\,kg \tag{178}$$

Example 4. Assuming there are no other gravitational influences besides the Earth, compute the new minimum *natural relativistic* gravitational mass $M_{\min}$ of an object at $y_1 = \infty$.

So, given,

Mass of object $M_{y_0} = 1.0\,kg$

Object on Earth's surface $y_0 = 6.3781 \times 10^6\,m$

Speed of light $c = 2.99792458 \times 10^8\,m/\sec$

Gravitational constant $G = 6.67260 \times 10^{-11}\,N\,m^2/kg^2$

Mass of the Earth $M_E = 5.9787 \times 10^{24}\,kg$

The minimum gravitational mass $M_{\min}$ at $y_1 = \infty$ is,

$$M_{\min} = M_{y_0}\, e^{\left(-\frac{G\,M_E}{y_0\,c^2}\right)} = (1.0\,kg)\, e^{\left(-\frac{(6.67260\times10^{-11}\,N\,m^2/kg^2)(5.9787\times10^{24}\,kg)}{(6.3781\times10^{6}\,m)(2.99792458\times10^{8}\,m/\sec)^2}\right)} \tag{179}$$

$$M_{\min} = 0.9999999993041\,kg \tag{180}$$

The gravitational mass of the object was reduced by,

$$M_{y_1} - M_{y_0} = (0.9999999993041\,kg) - (1.0\,kg) = -6.959\times10^{-10}\,kg \tag{181}$$

HOW GRAVITY AFFECTS THE VOLUME OF OBJECTS

An object of volume $\mathcal{V}_y$ (length $\mathcal{L}_y$, width $\mathcal{W}_y$, and height $\mathcal{H}_y$) contracts as a function of position $+\Delta y$ within gravity well g_y. Likewise, the same the volume $\mathcal{V}_y$ dilates as a function of position $-\Delta y$ in the same gravity well.

So, the difference form of the gravitational RED SHIFT (or BLUE SHIFT) of an object of volume $\Delta\mathcal{V}/\mathcal{V}$ displaced $\Delta y = y_0 - y_1$ within a given gravity well g_y is,

$$\frac{\Delta\mathcal{L}}{\mathcal{L}} = \frac{\Delta\mathcal{W}}{\mathcal{W}} = \frac{\Delta\mathcal{H}}{\mathcal{H}} = -\frac{g_{y_1}\,y_1 - g_{y_0}\,y_0}{c^2} \tag{182}$$

The difference forms and exponential forms of the *natural relativistic* object of volume $\mathcal{V}_y$ at position $\pm\Delta y$ is,

$$\mathcal{L}_{y_1} = \mathcal{L}_{y_0} \mp \Delta\mathcal{L}_{y_0} = \mathcal{L}_{y_0}\left(1 \mp \frac{g_{y_1}\,y_1 - g_{y_0}\,y_0}{c^2}\right) = \mathcal{L}_{y_0}\, e^{\left(-\frac{g_{y_1}\,y_1 - g_{y_0}\,y_0}{c^2}\right)} \tag{183}$$

$$\mathcal{W}_{y_1} = \mathcal{W}_{y_0} \mp \Delta\mathcal{W}_{y_0} = \mathcal{W}_{y_0}\left(1 \mp \frac{g_{y_1}\,y_1 - g_{y_0}\,y_0}{c^2}\right) = \mathcal{W}_{y_0}\, e^{\left(-\frac{g_{y_1}\,y_1 - g_{y_0}\,y_0}{c^2}\right)} \tag{184}$$

$$\mathcal{H}_{y_1} = \mathcal{H}_{y_0} \mp \Delta\mathcal{H}_{y_0} = \mathcal{H}_{y_0}\left(1 \mp \frac{g_{y_1}\,y_1 - g_{y_0}\,y_0}{c^2}\right) = \mathcal{H}_{y_0}\, e^{\left(-\frac{g_{y_1}\,y_1 - g_{y_0}\,y_0}{c^2}\right)} \tag{185}$$

HOW VELOCITY AFFECTS THE VOLUME OF OBJECTS

An object of volume $\mathcal{V}_v$ (length $\mathcal{L}_v$, width $\mathcal{W}_v$, and height $\mathcal{H}_v$) contracts moving at a *real* velocity v_x. Likewise, the same object of volume $\mathcal{V}_v$ dilates moving at a *complex* velocity $j\,v_x$.

So, the difference form of the inertial RED SHIFT (or BLUE SHIFT) of an object of volume $\Delta\mathcal{V}/\mathcal{V}$ moving at a *real* velocity v_x or a *complex* velocity $j\,v_x$ is,

$$\frac{\Delta\mathcal{L}}{\mathcal{L}} = \frac{\Delta\mathcal{W}}{\mathcal{W}} = \frac{\Delta\mathcal{H}}{\mathcal{H}} = -\frac{{v_x}^2}{2\,c^2} \tag{186}$$

The difference forms and exponential forms of the *special relativistic* object of volume $\mathcal{V}_v$ moving at a *real* velocity v_x or a *complex* velocity $j\,v_x$ is,

$$\mathcal{L}_v = \mathcal{L}_0 \mp \Delta\mathcal{L}_0 = \mathcal{L}_0\left(1 \mp \frac{{v_x}^2}{2\,c^2}\right) = \mathcal{L}_0\, e^{\left(-\frac{{v_x}^2}{2\,c^2}\right)} \quad (187)$$

$$\mathcal{W}_v = \mathcal{W}_0 \mp \Delta\mathcal{W}_0 = \mathcal{W}_0\left(1 \mp \frac{{v_x}^2}{2\,c^2}\right) = \mathcal{W}_0\, e^{\left(-\frac{{v_x}^2}{2\,c^2}\right)} \quad (188)$$

$$\mathcal{H}_v = \mathcal{H}_0 \mp \Delta\mathcal{H}_0 = \mathcal{H}_0\left(1 \mp \frac{{v_x}^2}{2\,c^2}\right) = \mathcal{H}_0\, e^{\left(-\frac{{v_x}^2}{2\,c^2}\right)} \quad (189)$$

HOW GRAVITY AFFECTS THE FREQUENCY OF TIME

A mechanical oscillator vibrating at a frequency f_y contracts (i.e., slows down) as a function of position $+\Delta y$ within gravity well g_y. Likewise, the same mechanical oscillator vibrating at a frequency f_y dilates (i.e., speeds up) as a function of position $-\Delta y$ in the same gravity well.

So, the difference form of the gravitational RED SHIFT (or BLUE SHIFT) of an oscillator vibrating at a frequency $\Delta f / f$ displaced $\Delta y = y_0 - y_1$ within a given gravity well g_y is,

$$\frac{\Delta f}{f} = -\frac{g_{y_1}\, y_1 - g_{y_0}\, y_0}{c^2} \quad (190)$$

The difference form and exponential form of the *natural relativistic* frequency f_y of an oscillator at position $\pm\Delta y$ is,

$$f_{y_1} = f_{y_0} \mp \Delta f_{y_0} = f_{y_0}\left(1 \mp \frac{g_{y_1}\, y_1 - g_{y_0}\, y_0}{c^2}\right) = f_{y_0}\, e^{\left(-\frac{g_{y_1}\, y_1 - g_{y_0}\, y_0}{c^2}\right)} \quad (191)$$

HOW VELOCITY AFFECTS THE FREQUENCY OF TIME

A mechanical oscillator vibrating at a frequency f_v contracts (i.e., slows down) while moving at a *real* velocity v_x. Likewise, the same mechanical oscillator vibrating at a frequency f_v dilates (i.e., speeds up) while moving at a *complex* velocity $j\,v_x$.

So, the difference form of the inertial RED SHIFT (or BLUE SHIFT) of an oscillator vibrating at a frequency $\Delta f / f$ while moving at a *real* velocity v_x or a *complex* velocity $j\,v_x$ is,

$$\frac{\Delta f}{f} = -\frac{{v_x}^2}{2\,c^2} \quad (192)$$

The difference form and exponential form of the *special relativistic* frequency f_v of an oscillator moving at a *real* velocity v_x or a *complex* velocity $j\,v_x$ is,

$$f_v = f_0 \mp \Delta f_0 = f_0\left(1 \mp \frac{v_x^{\,2}}{2c^2}\right) = f_0\, e^{\left(-\frac{v_x^{\,2}}{2c^2}\right)} \tag{193}$$

HOW GRAVITY AFFECTS AN INTERVAL OF TIME

A mechanical oscillator vibrating for an interval of time t_y contracts (i.e., slows down) as a function of position $+\Delta y$ within gravity well g_y. Likewise, the same mechanical oscillator vibrating for an interval of time t_y dilates (i.e., speeds up) as a function of position $-\Delta y$ in the same gravity well.

So, the difference form of the gravitational RED SHIFT (or BLUE SHIFT) of an oscillator vibrating for an interval of time $\Delta t/t$ displaced $\Delta y = y_0 - y_1$ within a given gravity well g_y is,

$$\frac{\Delta t}{t} = -\frac{g_{y_1}\, y_1 - g_{y_0}\, y_0}{c^2} \tag{194}$$

The difference form and exponential form of the *natural relativistic* time interval t_y of an oscillator at position $\pm\Delta y$ is,

$$t_{y_1} = t_{y_0} \mp \Delta t_{y_0} = t_{y_0}\left(1 \mp \frac{g_{y_1}\, y_1 - g_{y_0}\, y_0}{c^2}\right) = t_{y_0}\, e^{\left(-\frac{g_{y_1}\, y_1 - g_{y_0}\, y_0}{c^2}\right)} \tag{195}$$

HOW VELOCITY AFFECTS AN INTERVAL OF TIME

A mechanical oscillator vibrating for an interval of time t_v contracts (i.e., slows down) while moving at a *real* velocity v_x. Likewise, the same mechanical oscillator vibrating for an interval of time t_v dilates (i.e., speeds up) while moving at a *complex* velocity $j\,v_x$.

So, the difference form of the inertial RED SHIFT (or BLUE SHIFT) of an oscillator vibrating for an interval of time $\Delta t/t$ while moving at a *real* velocity v_x or a *complex* velocity $j\,v_x$ is,

$$\frac{\Delta t}{t} = -\frac{v_x^{\,2}}{2c^2} \tag{196}$$

The difference form and exponential form of the *special relativistic* time interval t_v of an oscillator moving at a *real* velocity v_x or a *complex* velocity $j\,v_x$ is,

$$t_v = t_0 \mp \Delta t_0 = t_0\left(1 \mp \frac{v_x^{\,2}}{2c^2}\right) = t_0\, e^{\left(-\frac{v_x^{\,2}}{2c^2}\right)} \tag{197}$$

HOW GRAVITY AFFECTS LINEAR MOMENTUM

The momentum p_y of an object of mass M_y moving at velocity v_x increases as a function of position $+\Delta y$ within gravity well g_y. Likewise, the momentum p_y of the same object decreases as a function of position $-\Delta y$ in the same gravity well,

$$p_y = M_y\, v_x \tag{198}$$

So, the difference form of the gravitational RED SHIFT (or BLUE SHIFT) of momentum $\Delta p/p$ of an object displaced $\Delta y = y_0 - y_1$ within a given gravity well g_y is,

$$\frac{\Delta p}{p} = \frac{g_{y_1}\, y_1 - g_{y_0}\, y_0}{c^2} \tag{199}$$

The difference form and exponential form of the *natural relativistic* momentum p_y at position $\pm\Delta y$ is,

$$p_{y_1} = p_{y_0} \pm \Delta p_{y_0} = p_{y_0}\left(1 \pm \frac{g_{y_1}\, y_1 - g_{y_0}\, y_0}{c^2}\right) = p_{y_0}\, e^{\left(\frac{g_{y_1}\, y_1 - g_{y_0}\, y_0}{c^2}\right)} \tag{200}$$

HOW VELOCITY AFFECTS LINEAR MOMENTUM

The momentum p_v of an object of mass M_0 increases moving at a *real* velocity v_x. Likewise, the momentum p_v of the same object decreases moving at a *complex* velocity $j\,v_x$,

$$p_v = M_0\, v_x \tag{201}$$

So, the difference form of the inertial RED SHIFT (or BLUE SHIFT) of momentum $\Delta p/p$ of an object moving at a *real* velocity v_x or a *complex* velocity $j\,v_x$ is,

$$\frac{\Delta p}{p} = \frac{{v_x}^2}{2\,c^2} \tag{202}$$

The difference form and exponential form of the *special relativistic* momentum p_v of an object moving at a *real* velocity v_x or a *complex* velocity $j\,v_x$ is,

$$p_v = p_0 \pm \Delta p_0 = p_0\left(1 \pm \frac{{v_x}^2}{2\,c^2}\right) = p_0\, e^{\left(\frac{{v_x}^2}{2\,c^2}\right)} \tag{203}$$

HOW GRAVITY AFFECTS ANGULAR MOMENTUM

The angular momentum S_y of an object of mass M_y moving at velocity v_x with a radius r is **invariant** as a function of position $+\Delta y$ within gravity well g_y. Likewise, the angular momentum S_y of the same object is **invariant** as a function of position $-\Delta y$ in the same gravity well,

$$S_y = M_y\, v_x\, r \tag{204}$$

So, the difference form of the gravitational RED SHIFT (or BLUE SHIFT) of angular momentum $\Delta S/S$ of an object displaced $\Delta y = y_0 - y_1$ within a given gravity well g_y is,

$$\frac{\Delta S}{S} = 0 \tag{205}$$

The difference form and exponential form of the *natural relativistic* angular momentum S_y at position $\pm\Delta y$ is,

$$S_{y_1} = S_{y_0} \tag{206}$$

HOW VELOCITY AFFECTS ANGULAR MOMENTUM

The angular momentum S_v of an object of mass M_0 is **invariant** moving at a *real* velocity v_x with a radius r. Likewise, the angular momentum S_v of the same object is **invariant** moving at a *complex* velocity $j\,v_x$,

$$S_v = M_0\, v_x\, r \tag{207}$$

So, the difference form of the inertial RED SHIFT (or BLUE SHIFT) of angular momentum $\Delta S/S$ of an object moving at a *real* velocity v_x or a *complex* velocity $j\,v_x$ is,

$$\frac{\Delta S}{S} = 0 \tag{208}$$

The difference form and exponential form of the *special relativistic* angular momentum S_v of an object moving at a *real* velocity v_x or a *complex* velocity $j\,v_x$ is,

$$S_v = S_0 \tag{209}$$

THE SPACE-TIME MEDIA OR THE AETHER

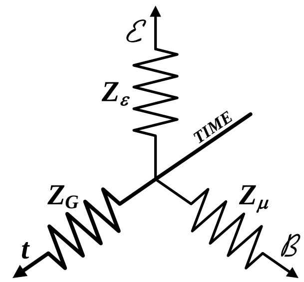


FIGURE 14. The space-time media or aether.

According to Puthoff (1996) and Puthoff, Little and Ibison (2002), the vacuum is described as having magnetic permeability μ_0 and dielectric permittivity ε_0, and acts to impede the propagation of light and the motion of matter. Direct modification of these components changes the nature of light and matter.

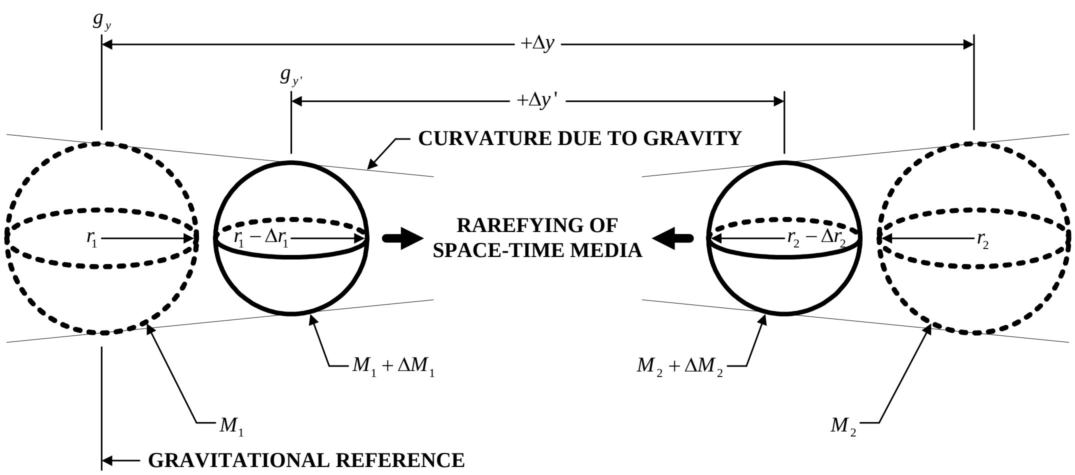


FIGURE 15. Two similar objects undergoing natural universal mass attraction.

The "active vacuum" of space, or space-time media (i.e., the aether) is composed of uncondensed *relativistic* mass. An object M_1 made of matter (i.e., atoms) and given a GRAVITATIONAL REFERENCE point undergoes universal mass attraction (i.e., gravitational free fall) with another object M_2. Both objects acquire *relativistic* mass by a natural means from the surrounding space-time media as a function of displacement $+\Delta y$ between the two objects. This media condenses onto both objects as more *relativistic* mass, thereby increasing their total mass $M_n + \Delta M_n$, inductance $L_n + \Delta L_n$, and capacitance $C_n + \Delta C_n$. This action changes the *relativistic* momentum of both objects resulting with increasing force of attraction. The space-time media between these objects rarefy or *relativistic* mass condenses out of the media thereby affecting both the magnetic permeability μ_0 and the dielectric permittivity ε_0 of free space. This rarefaction of media is referred to as a gravity well, and as a consequence, causes the volume of space occupied by both objects and the space between them to be reduced. The space-time media in a rarefying state means gravity between these objects is increasing, which causes light passing near these objects to amplify in energy $\mathcal{E}_{\lambda y}$ and increase in frequency $f_{\lambda y}$ as proven by the Pound and Rebka experiment (1964). This behavior of space-time media acts as an impedance upon the natural motion of matter and the propagation of light.

THE GRAVITATIONAL COUPLING OF AN ELECTROMAGNETIC WAVE

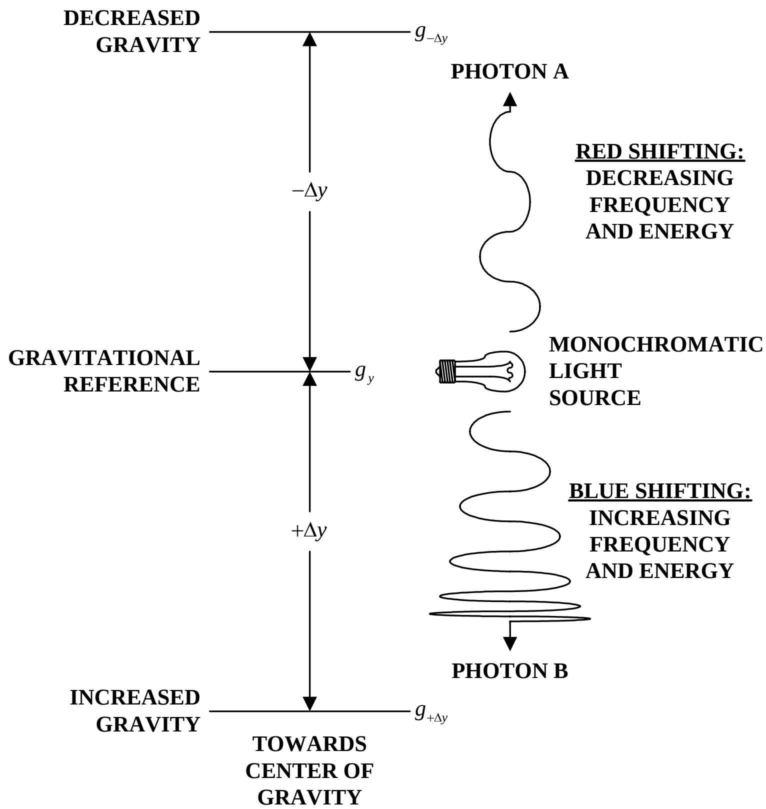


FIGURE 16. Electromagnetic waves propagating within a given gravity well.

Shown above is the BLUE SHIFTING of an electromagnetic wave due to gravity. *Relative* to a GRAVITATIONAL REFERENCE point or equipotential surface of gravity within a given gravity well g_y, PHOTON A decreases in energy $\mathcal{E}_{\lambda y}$ and frequency $f_{\lambda y}$ as it propagates through decreasing gravity $g_{-\Delta y}$. Likewise, PHOTON B increases in energy $\mathcal{E}_{\lambda y}$ and frequency $f_{\lambda y}$ as it propagates through increasing gravity $g_{+\Delta y}$. This effect was demonstrated in the Pound, Rebka and Snider experiment, which used Mossbauer spectroscopy to measure the electromagnetic gravitational RED SHIFT (or BLUE SHIFT) of $14.4\,keV$ gamma rays emitted from Fe^{57} through a vertical distance of $22.6\,m$. Using the difference form of fluctuating energy $\Delta\mathcal{E}_{\lambda}$ of an electromagnetic wave propagating through gravity well g_y and the gamma rays emitted upward, the RED SHIFT was within one percent (1%) of this result,

$$SHIFT = \frac{\Delta\mathcal{E}_{\lambda}}{\mathcal{E}_{\lambda}} = \frac{g_{y_1}\; y_1 - g_{y_0}\; y_0}{c^2} \tag{210}$$

$$SHIFT = \frac{(9.806581\,m/\sec^2)(6.3781226\times10^6\,m) - \left(9.806650\,m/\sec^2\right)\left(6.3781000\times10^6\,m\right)}{(2.99792458\times10^8\;m/\sec)^2} \tag{211}$$

$$SHIFT = -2.465961\times10^{-15} \tag{212}$$

And with the gamma rays emitted downward, the BLUE SHIFT was,

$$SHIFT = \frac{\left(9.806650\, m/\sec^2\right)\left(6.3781000\times10^6\, m\right) - (9.806581\, m/\sec^2)(6.3781226\times10^6\, m)}{(2.99792458\times10^8\, m/\sec)^2} \tag{213}$$

$$SHIFT = 2.465961\times10^{-15} \tag{214}$$

The difference form and exponential form of the *natural relativistic* electromagnetic energy $\mathcal{E}_{\lambda y}$ at position $\pm\Delta y$ is,

$$\mathcal{E}_{\lambda y_1} = \mathcal{E}_{\lambda y_0} \pm \Delta\mathcal{E}_{\lambda y_0} = \mathcal{E}_{\lambda y_0}\left(1 \pm \frac{g_{y_1}\, y_1 - g_{y_0}\, y_0}{c^2}\right) = \mathcal{E}_{\lambda y_0}\, e^{\left(\frac{g_{y_1}\, y_1 - g_{y_0}\, y_0}{c^2}\right)} \tag{215}$$

Since the energy $\mathcal{E}_{\lambda}$ of a single photon is,

$$\mathcal{E}_{\lambda} = h\, f_{\lambda} \tag{216}$$

Then, the Planck's constant h is,

$$h = \frac{\mathcal{E}_{\lambda}}{f_{\lambda}} \tag{217}$$

The difference form of the gravitational RED SHIFT (or BLUE SHIFT) of frequency $\Delta f_{\lambda} / f_{\lambda}$ of an electromagnetic wave propagating $\Delta y = y_0 - y_1$ within a given gravity well g_y is,

$$\frac{\Delta f_{\lambda}}{f_{\lambda}} = \frac{g_{y_1}\, y_1 - g_{y_0}\, y_0}{c^2} \tag{218}$$

The difference form and exponential form of the *natural relativistic* electromagnetic frequency $f_{\lambda y}$ at position $\pm\Delta y$ is,

$$f_{\lambda y_1} = f_{\lambda y_0} \pm \Delta f_{\lambda y_0} = f_{\lambda y_0}\left(1 \pm \frac{g_{y_1}\, y_1 - g_{y_0}\, y_0}{c^2}\right) = f_{\lambda y_0}\, e^{\left(\frac{g_{y_1}\, y_1 - g_{y_0}\, y_0}{c^2}\right)} \tag{219}$$

So, the *natural relativistic* Planck's constant h_y ranging from position $-\Delta y$ to $+\Delta y$ evaluates to unity gain or,

$$h_{y_1} = \frac{\mathcal{E}_{\lambda y_1}}{f_{\lambda y_1}} = \frac{\mathcal{E}_{\lambda y_0}\, e^{\left(\frac{g_{y_1}\, y_1 - g_{y_0}\, y_0}{c^2}\right)}}{f_{\lambda y_0}\, e^{\left(\frac{g_{y_1}\, y_1 - g_{y_0}\, y_0}{c^2}\right)}} = \frac{\mathcal{E}_{\lambda}}{f_{\lambda}} = h \tag{220}$$

Therefore, the *natural relativistic* Planck's constant h_y is **invariant between equipotential surfaces of gravity** or,

$$h_y = 6.6260755\times10^{-34}\, Joule\cdot\sec \tag{221}$$

The speed of light c is,

$$c = \lambda f_\lambda \tag{222}$$

The difference form of the gravitational RED SHIFT (or BLUE SHIFT) of the wavelength $\Delta\lambda/\lambda$ of an electromagnetic wave displaced $\Delta y = y_0 - y_1$ within a given gravity well g_y is,

$$\frac{\Delta\lambda}{\lambda} = -\frac{g_{y_1}\, y_1 - g_{y_0}\, y_0}{c^2} \tag{223}$$

The *natural relativistic* wavelength λ_y at position $\pm\Delta y$ is,

$$\lambda_{y_1} = \lambda_{y_0} \mp \Delta\lambda_{y_0} = \lambda_{y_0}\left(1 \mp \frac{g_{y_1}\, y_1 - g_{y_0}\, y_0}{c^2}\right) = \lambda_{y_0}\, e^{\left(-\frac{g_{y_1}\, y_1 - g_{y_0}\, y_0}{c^2}\right)} \tag{224}$$

So, *natural relativistic* the speed of light c_y ranging from position $-\Delta y$ to $+\Delta y$ evaluates to unity gain or,

$$c_{y_1} = \lambda_{y_1}\, f_{\lambda y_1} = \lambda_{y_0}\, e^{\left(-\frac{g_{y_1}\, y_1 - g_{y_0}\, y_0}{c^2}\right)}\, f_{\lambda y_0}\, e^{\left(\frac{g_{y_1}\, y_1 - g_{y_0}\, y_0}{c^2}\right)} = \lambda\, f_\lambda = c \tag{225}$$

Therefore, the *natural relativistic* speed of light c_y is **invariant between equipotential surfaces of gravity** or,

$$c_y = 2.99792458 \times 10^8\ m/\sec \tag{226}$$

HOW GRAVITY AFFECTS THE PERMEABLILITY OF SPACE-TIME MEDIA

The permeability μ_0 of space-time media is given as,

$$\mu_0 = 4\pi \times 10^{-7}\ H/m \tag{227}$$

The difference form of the gravitational RED SHIFT (or BLUE SHIFT) of inductance $\Delta L/L$ displaced $\Delta y = y_0 - y_1$ within a given gravity well g_y is,

$$\frac{\Delta L}{L} = -\frac{g_{y_1}\, y_1 - g_{y_0}\, y_0}{c^2} \tag{228}$$

The inductance L_y of space-time media at position $\pm\Delta y$ is,

$$L_{y_1} = L_{y_0} \mp \Delta L_{y_0} = L_{y_0}\left(1 \mp \frac{g_{y_1}\, y_1 - g_{y_0}\, y_0}{c^2}\right) = L_{y_0}\, e^{\left(-\frac{g_{y_1}\, y_1 - g_{y_0}\, y_0}{c^2}\right)} \tag{229}$$

The difference form of the gravitational RED SHIFT (or BLUE SHIFT) of length $\Delta\mathcal{L}/\mathcal{L}$ displaced $\Delta y = y_0 - y_1$ within a given gravity well g_y is,

$$\frac{\Delta\mathcal{L}}{\mathcal{L}} = -\frac{g_{y_1}\ y_1 - g_{y_0}\ y_0}{c^2} \tag{230}$$

The length $\mathcal{L}_y$ of space-time media at position $\pm\Delta y$ is,

$$\mathcal{L}_{y_1} = \mathcal{L}_{y_0} \mp \Delta\mathcal{L}_{y_0} = \mathcal{L}_{y_0}\left(1 \mp \frac{g_{y_1}\ y_1 - g_{y_0}\ y_0}{c^2}\right) = \mathcal{L}_{y_0}\ e^{\left(-\frac{g_{y_1}\ y_1 - g_{y_0} y_0}{c^2}\right)} \tag{231}$$

So, the *natural relativistic* permeability μ_y of space-time media ranging from position $-\Delta y$ to $+\Delta y$ evaluates to unity gain or,

$$\mu_{y_1} = \frac{L_{y_1}}{\mathcal{L}_{y_1}} = \frac{L_{y_0}\ e^{\left(-\frac{g_{y_1}\ y_1 - g_{y_0} y_0}{c^2}\right)}}{\mathcal{L}_{y_0}\ e^{\left(-\frac{g_{y_1}\ y_1 - g_{y_0} y_0}{c^2}\right)}} = \frac{L}{\mathcal{L}} = \mu_0 \tag{232}$$

Therefore, the *natural relativistic* permeability μ_y of space-time media is **invariant between equipotential surfaces of gravity** or,

$$\mu_y = 4\pi \times 10^{-7}\ H/m \tag{233}$$

HOW GRAVITY AFFECTS THE PERMITTIVITY OF SPACE-TIME MEDIA

The permittivity ε_0 of space-time media is given as,

$$\varepsilon_0 = 8.85419 \times 10^{-12}\ F/m \tag{234}$$

The difference form of the gravitational RED SHIFT (or BLUE SHIFT) of capacitance $\Delta C/C$ displaced $\Delta y = y_0 - y_1$ within a given gravity well g_y is,

$$\frac{\Delta C}{C} = -\frac{g_{y_1}\ y_1 - g_{y_0}\ y_0}{c^2} \tag{235}$$

The capacitance C_y of space-time media at position $\pm\Delta y$ is,

$$C_{y_1} = C_{y_0} \mp \Delta C_{y_0} = C_{y_0}\left(1 \mp \frac{g_{y_1}\ y_1 - g_{y_0}\ y_0}{c^2}\right) = C_{y_0}\ e^{\left(-\frac{g_{y_1}\ y_1 - g_{y_0}\ y_0}{c^2}\right)} \tag{236}$$

The difference form of the gravitational RED SHIFT (or BLUE SHIFT) of length $\Delta\mathcal{L}/\mathcal{L}$ displaced $\Delta y = y_0 - y_1$ within a given gravity well g_y is,

$$\frac{\Delta\mathcal{L}}{\mathcal{L}} = -\frac{g_{y_1}\, y_1 - g_{y_0}\, y_0}{c^2} \tag{237}$$

The length $\mathcal{L}_y$ of space-time media at position $\pm\Delta y$ is,

$$\mathcal{L}_{y_1} = \mathcal{L}_{y_0} \mp \Delta\mathcal{L}_{y_0} = \mathcal{L}_{y_0}\left(1 \mp \frac{g_{y_1}\, y_1 - g_{y_0}\, y_0}{c^2}\right) = \mathcal{L}_{y_0}\, e^{\left(-\frac{g_{y_1}\, y_1 - g_{y_0} y_0}{c^2}\right)} \tag{238}$$

So, the *natural relativistic* permittivity ε_y of space-time media ranging from position $-\Delta y$ to $+\Delta y$ evaluates to unity gain or,

$$\varepsilon_{y_1} = \frac{C_{y_1}}{\mathcal{L}_{y_1}} = \frac{C_{y_0}\, e^{\left(-\frac{g_{y_1}\, y_1 - g_{y_0} y_0}{c^2}\right)}}{\mathcal{L}_{y_0}\, e^{\left(-\frac{g_{y_1}\, y_1 - g_{y_0} y_0}{c^2}\right)}} = \frac{C}{\mathcal{L}} = \varepsilon_0 \tag{239}$$

Therefore, the *natural relativistic* permittivity ε_y of space-time media is **invariant between equipotential surfaces of gravity** or,

$$\varepsilon_y = 8.85419 \times 10^{-12}\ F/m \tag{240}$$

HOW GRAVITY AFFECTS THE VIRTUAL RESISTANCE OF SPACE-TIME MEDIA

The space-time media has virtual resistance or impedance Z_0, and therefore, isn't capable of absorbing or dissipating electromagnetic energy. Its REAL resistance is infinite or, $R_0 = \infty$. This media serves to impede the propagation of light and the motion of matter and is calculated as,

$$Z_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} \tag{241}$$

Since it has been shown the permeability μ_y and permittivity ε_y of space-time media are invariant between equipotential surfaces of gravity, it follows the *natural relativistic* impedance Z_y of space-time media ranging from position $-\Delta y$ to $+\Delta y$ evaluates to unity gain or,

$$Z_{y_1} = \sqrt{\frac{\mu_{y_1}}{\varepsilon_{y_1}}} = Z_0 \tag{242}$$

Therefore, the *natural relativistic* impedance Z_y of space-time media is **invariant between equipotential surfaces of gravity** or,

$$Z_y = 376.730\,\Omega \tag{243}$$

HOW GRAVITY AFFECTS THE SPEED OF LIGHT

The speed of light c between space-time media is,

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} \tag{244}$$

Since it has been shown the permeability μ_y and permittivity ε_y of space-time media are invariant between equipotential surfaces of gravity, it follows the *natural relativistic* speed of light c_y through space-time media ranging from position $-\Delta y$ to $+\Delta y$ evaluates to unity gain or,

$$c_{y_1} = \frac{1}{\sqrt{\mu_{y_1} \varepsilon_{y_1}}} = c \tag{245}$$

Therefore, the *natural relativistic* speed of light c_y through space-time media is **invariant between equipotential surfaces of gravity** or,

$$c_y = 2.997924 \times 10^8 \; m/\sec \tag{246}$$

HOW GRAVITY AFFECTS BOLTZMANN'S CONSTANT

The Boltzmann's Constant k is given as,

$$k = \frac{R}{N_0} \tag{247}$$

Since the Ideal Gas Constant R_y and Avogadro's Number N_y are invariant between equipotential surfaces of gravity, it follows the *natural relativistic* Boltzmann's Constant k_y ranging from position $-\Delta y$ to $+\Delta y$ evaluates to unity gain or,

$$k_{y_1} = \frac{R_{y_1}}{N_{y_1}} = k \tag{248}$$

Therefore, the *natural relativistic* Boltzmann's Constant k_y is **invariant between equipotential surfaces of gravity** or,

$$k_y = 1.380658 \times 10^{-23} \; Joules/{}^\circ K \tag{249}$$

HOW GRAVITY AFFECTS AN ELECTRIC CHARGE

A fundamental electric charge q is given as,

$$q = \frac{\mathsf{f}}{\mathcal{E}} \tag{250}$$

The electric force f_y increases with the square of a decreasing distance, and the electric field $\mathcal{E}_y$ also increases with the square of a decreasing distance at position $+dy$. Likewise, the electric force f_y decreases with the square of a increasing distance, and the electric field $\mathcal{E}_y$ also decreases with the square of a increasing distance at position $-dy$, it follows the *natural relativistic* electric charge q_y ranging from position $-\Delta y$ to $+\Delta y$ evaluates to unity gain or,

$$q_{y_1} = \frac{\mathsf{f}_{y_1}}{\mathcal{E}_{y_1}} = q \tag{251}$$

Therefore, the *natural relativistic* electric charge q_y is **invariant between equipotential surfaces of gravity** or,

$$q_y = 1.60217733 \times 10^{-19}\, Coul \tag{252}$$

HOW GRAVITY AFFECTS THE FINE STRUCTURE CONSTANT

The Fine Structure Constant α is,

$$\alpha = \frac{q^2}{2\,\varepsilon_0\, h\, c} \tag{253}$$

Since an electric charge q_y, the speed of light c_y, the permittivity ε_y, and Planck's constant h_y are invariant between equipotential surfaces of gravity, it follows the *natural relativistic* Fine Structure Constant α_y ranging from position $-\Delta y$ to $+\Delta y$ evaluates to unity gain or,

$$\alpha_{y_1} = \frac{{q_{y_1}}^2}{2\,\varepsilon_{y_1}\, h_{y_1} c_{y_1}} = \alpha \tag{254}$$

Therefore, the *natural relativistic* Fine Structure Constant α_y is **invariant between equipotential surfaces of gravity** or,

$$\alpha_y = 7.29738 \times 10^{-3} = \frac{1}{137.0356} \tag{255}$$

A TYPICAL ELECTROMAGNETIC WAVE

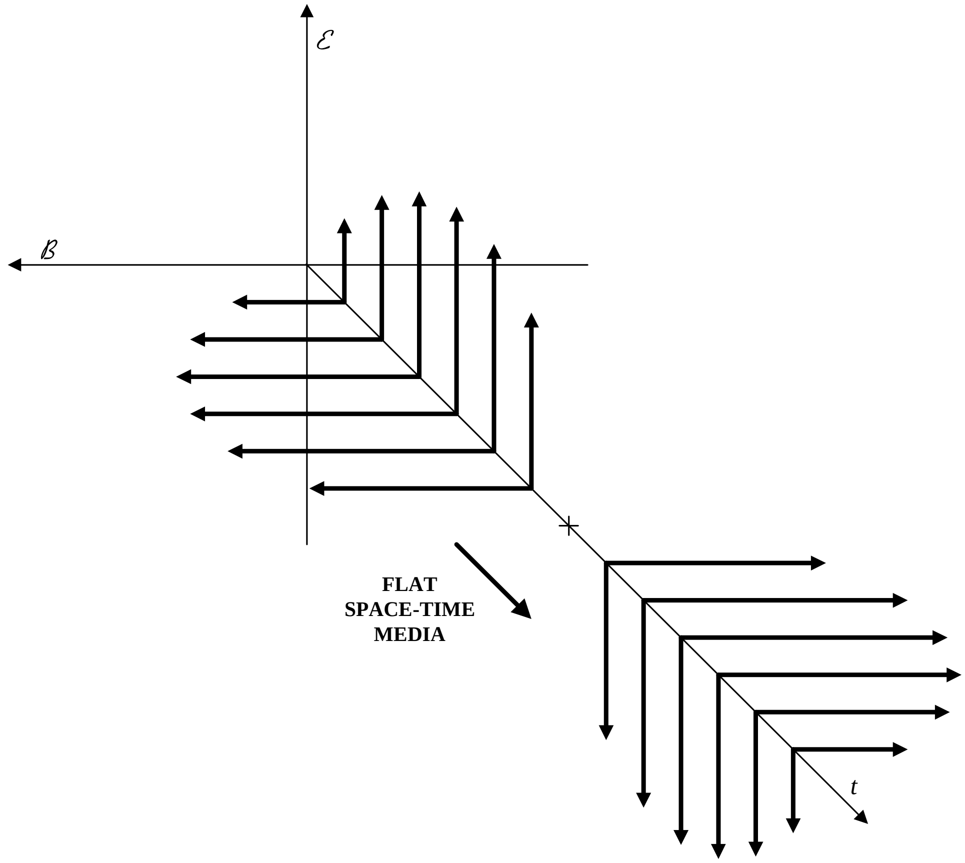


FIGURE 17. Propagation of electromagnetic wave in flat space-time.

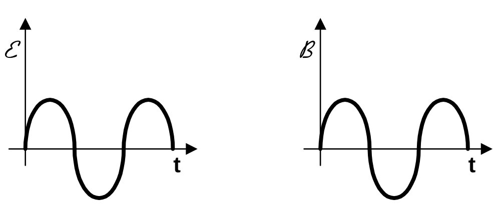


FIGURE 18. Typical ℬ and ℰ Fields.

GRAVITATIONAL BLUE SHIFTING OF AN ELECTROMAGNETIC WAVE

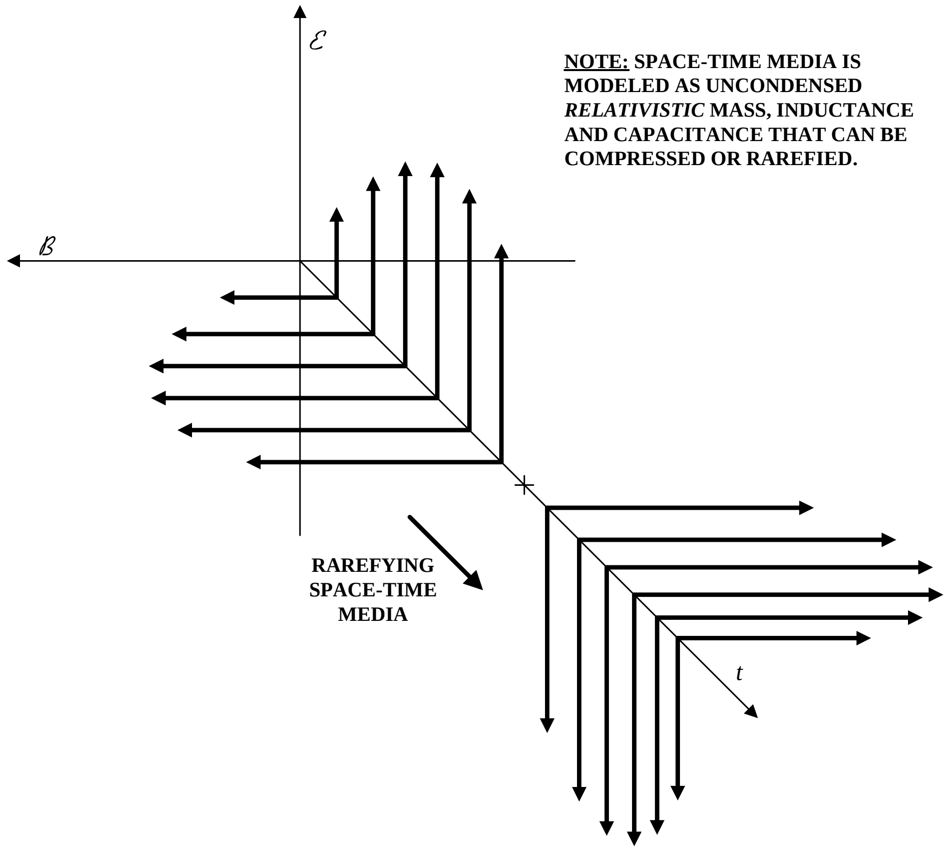


FIGURE 19. Propagation of electromagnetic wave in rarefied space-time.

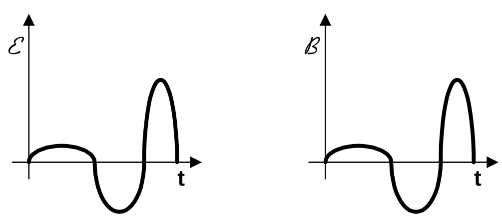


FIGURE 20. Increasing magnitude and frequency of ℬ and ℰ Fields by gravitational function.

GRAVITATIONAL RED SHIFTING OF AN ELECTROMAGNETIC WAVE

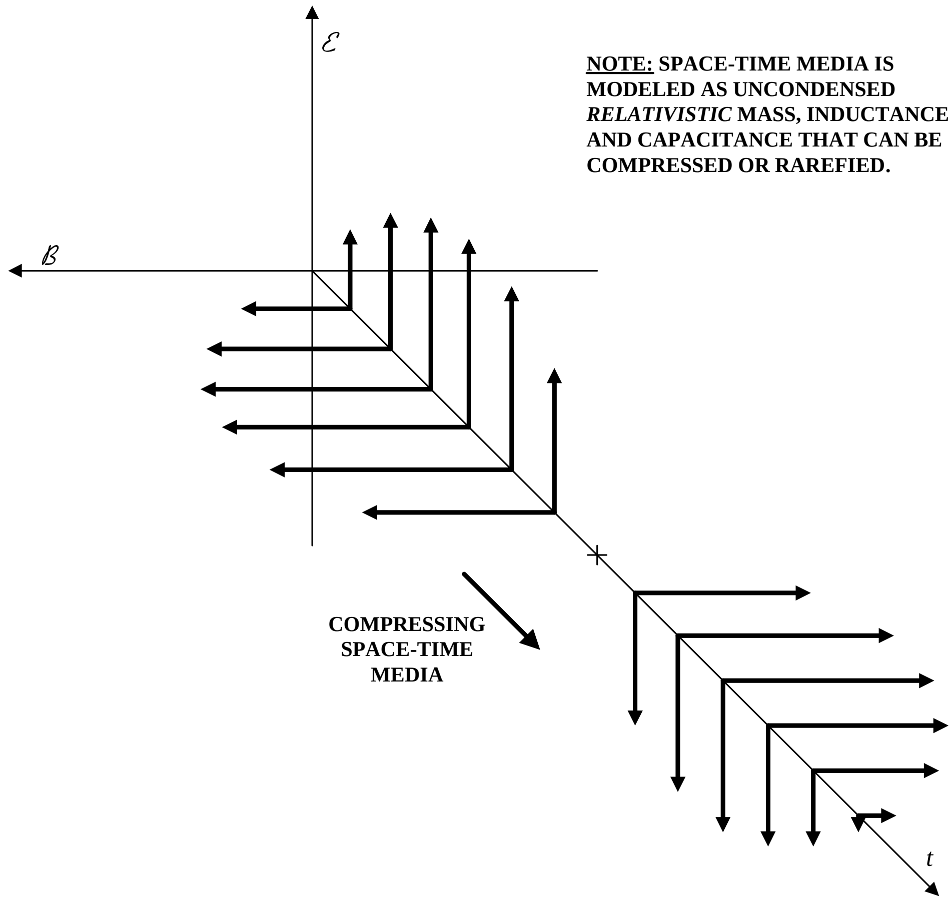


FIGURE 21. Propagation of electromagnetic wave in compressing space-time.

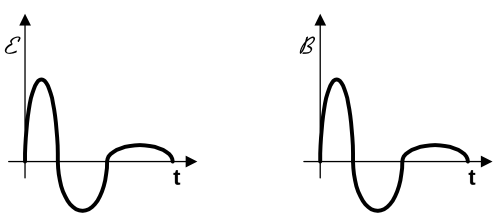


FIGURE 22. Decreasing magnitude and frequency of ℬ and ℰ Fields by gravitational function.

FIGURE 23. A Global Positioning Satellite.

Example 5. A Global Positioning Satellite (GPS) transmits an electromagnetic signal at a frequency $f_{\lambda SAT}$ of $\sim 10.23\,MHz$ down to the Earth from an altitude of $20{,}186.8\,km$, and has an orbital velocity of $3.874\,km/\sec$. The *natural relativistic* BLUE SHIFT due to gravity and the *special relativistic* RED SHIFT due to velocity changes the frequency of this transmitted signal. So, given the corrected transmitted frequency $f_{\lambda SAT}$, compute the BLUE SHIFT and RED SHIFT such that a ground-based receiver will read a signal $f_{\lambda RX}$ that is precisely $10230000.000000\,Hz$. The signal frequency of the satellite is adjustable down to $1\,\mu Hz$.

So, given,

Altitude of satellite $\Delta y = 20.1868 \times 10^{6}\,m$

Orbital velocity $v_x = 3.874 \times 10^{3}\,m/\sec$

Corrected frequency of satellite $f_{\lambda SAT} = 10229999.995444\,Hz$

Receiver located on surface of Earth $y_1 = 6.3781 \times 10^{6}\,m$

Speed of light $c = 2.99792458 \times 10^{8}\,m/\sec$

Gravitational constant $G = 6.67260 \times 10^{-11}\,N\,m^2/kg^2$

Mass of the Earth $M_E = 5.9787 \times 10^{24}\,kg$

The initial radius y_0 of the satellite above the Earth is,

$$y_0 = y_1 + \Delta y = \left(6.3781 \times 10^{6}\,m\right) + \left(20.1868 \times 10^{6}\,m\right) = 26.5649 \times 10^{6}\,m \qquad (256)$$

The acceleration due to gravity at altitude $y_0 = 26.5649 \times 10^6 m$ above the Earth is,

$$g_{y_0} = \frac{G M_E}{y_0^{\;2}} = \frac{\left(6.67260 \times 10^{-11} \, N\, m^2 / kg^2\right)\left(5.9787 \times 10^{24}\, kg\right)}{\left(26.5649 \times 10^6 m\right)^2} = 0.5653\, m/\sec^2 \tag{257}$$

The acceleration due to gravity at altitude $y_1 = 7.529 \times 10^6 m$ above the Earth is,

$$g_{y_1} = \frac{G M_E}{y_1^{\;2}} = \frac{\left(6.67260 \times 10^{-11} \, N\, m^2 / kg^2\right)\left(5.9787 \times 10^{24}\, kg\right)}{\left(6.3781 \times 10^6 m\right)^2} = 9.80665\, m/\sec^2 \tag{258}$$

Given the exponential solution of the *natural relativistic* frequency model, the gravitational BLUE SHIFTED frequency is,

$$f_{\lambda y_1} = f_{\lambda y_0}\, e^{\left(\frac{g_{y_1} y_1 - g_{y_0} y_0}{c^2}\right)} = \left(10229999.995444\, Hz\right) e^{\left(\frac{\left(9.80665 m/\sec^2\right)\left(6.3781\times 10^6 m\right) - \left(0.5653 m/\sec^2\right)\left(26.5649 \times 10^6 m\right)}{\left(2.99792458 \times 10^8 m/\sec\right)^2}\right)} \tag{259}$$

$$f_{\lambda y_1} = 10230000.000854\, Hz = f_\lambda \tag{260}$$

Given the exponential solution of the *special relativistic* frequency model, the RED SHIFTED frequency of the BLUE SHIFTED frequency computed above is,

$$f_{\lambda v} = f_\lambda\, e^{\left(-\frac{v_x^{\;2}}{2c^2}\right)} = \left(10230000.000854\, Hz\right) e^{\left(-\frac{\left(3.874 \times 10^3 m/\sec\right)^2}{2\left(2.99793 \times 10^8 m/\sec\right)^2}\right)} \tag{261}$$

$$f_{\lambda v} = 10230000.000000\, Hz = f_{\lambda RX} \tag{262}$$

So, a ground-based receiver will read a signal that is precisely $10230000.000000\, Hz$ with a satellite frequency $f_{\lambda SAT}$ given above.

FIGURE 24. A constellation of 24 Global Positioning Satellites (GPS) orbiting the Earth.

GRAVITOMAGNETIC THEORY

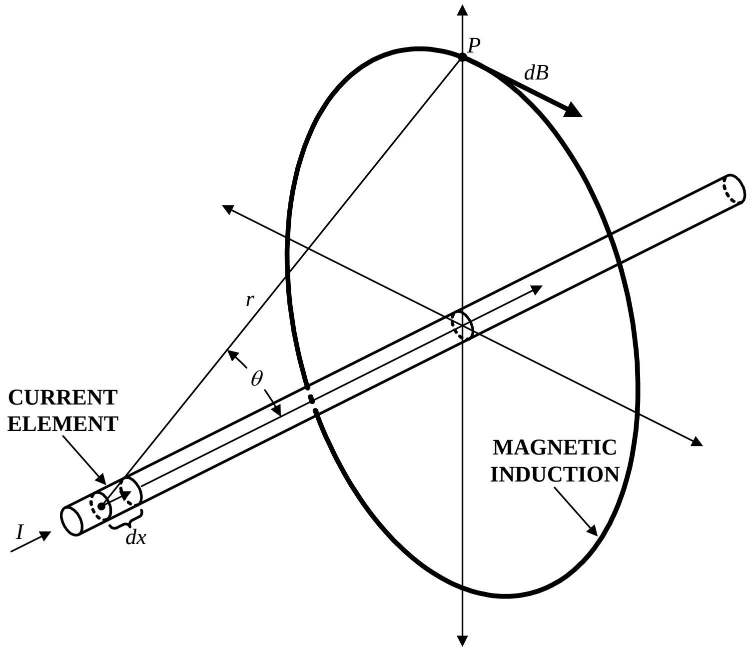


FIGURE 25. The magnetic induction produced by a positive current element.

A constant positive electric current I must create a stable magnetic field B around a wire. This stable field is due to the flow of electric current shown above. The change of magnetic induction dB at a fixed point P produced by a current element dx is calculated using the Biot-Savart's Law,

$$dB = \frac{\mu_0 I}{4\pi} \frac{dx \times r}{r^3} \tag{263}$$

Or,

$$dB = \frac{\mu_0 I}{4\pi} \frac{\sin(\theta) dx}{r^2} \tag{264}$$

Since charge q is quantized in a single electron e^- then, electric current I is defined as quantity N of charges e^- passing a fixed point per change of time dt or,

$$I = \dot{q} = \frac{dq}{dt} = \frac{d\left(N e^-\right)}{dt} \tag{265}$$

And velocity v_x of an electron passing a fixed point is defined as change of distance dx per change of time dt or,

$$v_x = \dot{x} = \frac{dx}{dt} \tag{266}$$

Then, the electric current I is redefined as,

$$I = v_x \frac{d\left(N\,e^-\right)}{dx} \tag{267}$$

So, the change of magnetic induction dB at a fixed point P produced by quantity N of charges e^- moving at velocity v_x is,

$$dB = \frac{\mu_0\,v_x}{4\,\pi}\frac{\sin\left(\theta\right)d\left(N\,e^-\right)}{r^2} \tag{268}$$

To find the magnetic induction B produced by a single electron at point P when $\theta = 90^\circ$ and $N = 1$, then integrate,

$$B = \int dB = \frac{\mu_0\,e^-\,v_x}{4\,\pi\,r^2} \tag{269}$$

The total energy density u_B of magnetic field B contained within volume $\mathcal{V}$ is,

$$u_B = \frac{U_B}{\mathcal{V}} = \frac{B^2}{2\,\mu_0} \tag{270}$$

Therefore, the total field energy U_B of magnetic field B contained within volume $\mathcal{V}$ is,

$$U_B = \frac{B^2}{2\,\mu_0}\mathcal{V} = \frac{\mu_0\left(e^-\right)^2 {v_x}^2}{32\,\pi^2\,r^4}\mathcal{V} \tag{271}$$

The change of magnetic field energy dU_B contained within a change of volume $d\mathcal{V}$ is,

$$dU_B = \frac{B^2}{2\,\mu_0}d\mathcal{V} = \frac{\mu_0\left(e^-\right)^2 {v_x}^2}{32\,\pi^2\,r^4}d\mathcal{V} \tag{272}$$

The total energy E_M contained within matter is,

$$E_M = M\,c^2 \tag{273}$$

Equate total magnetic field energy U_B to the total energy E_M contained within matter,

$$U_B = E_M \tag{274}$$

So, the change of magnetic field energy dU_B is,

$$dU_B = dM_B\,c^2 \tag{275}$$

Therefore, the fluctuating magnetic mass dM_B contained within a change of volume $d\mathcal{V}$ is,

$$dM_B = \frac{dU_B}{c^2} = \frac{\mu_0 \left(e^-\right)^2}{32\pi^2 r^4} \frac{v_x{}^2}{c^2} d\mathcal{V} \tag{276}$$

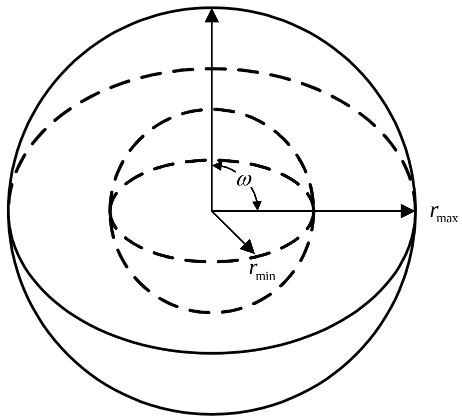


FIGURE 26. The volume $\mathcal{V}$ of an electron is modeled as a hollow spheroid.

Since the energy of an electron is finite, no field component can be present at its' center. So, the volume of an electron is modeled as a hollow spheroid,

$$\mathcal{V} = 2\pi \int_0^{\pi} \sin(\omega)\, d\omega \int_{r_{\min}}^{r_{\max}} r^2 dr = 4\pi \int_{r_{\min}}^{r_{\max}} r^2 dr \tag{277}$$

The fluctuating magnetic mass dM_B contained within a change of volume $d\mathcal{V}$ of a hollow spheroid is,

$$dM_B = \frac{\mu_0 \left(e^-\right)^2}{32\pi^2 r^4} \frac{v_x{}^2}{c^2} d\left(4\pi \int_{r_{\min}}^{r_{\max}} r^2 dr\right) = \frac{\mu_0 \left(e^-\right)^2}{8\pi} \frac{v_x{}^2}{c^2} d\left(\int_{r_{\min}}^{r_{\max}} \frac{1}{r^2} dr\right) \tag{278}$$

Given the radius of a fluctuating magnetic mass dM_e ranging from a classic electron radius r_e to infinity, or $r_e \le r \le \infty$, the derivative form of a moving electron is,

$$dM_B = \frac{\mu_0 \left(e^-\right)^2}{8\pi} \frac{v_x{}^2}{c^2} \int_{r_e}^{\infty} \frac{1}{r^2} dr \tag{279}$$

The fluctuating magnetic mass dM_B is,

$$dM_B = \frac{\mu_0 \left(e^-\right)^2}{8\pi\, r_e} \frac{v_x{}^2}{c^2} \tag{280}$$

So, given the rest mass of an electron M_e, the difference form of the *special relativistic* mass M_{ev} of an electron moving at velocity v_x is

$$M_{ev} = \gamma_{SR} M_e = M_e \pm dM_e = M_e \left(1 \pm \frac{v_x^{\ 2}}{2c^2}\right) \tag{281}$$

By equating the fluctuating magnetic mass dM_B to the *special relativistic* mass dM_e,

$$\frac{\mu_0 \left(e^-\right)^2}{8\pi r_e} \frac{v_x^{\ 2}}{c^2} \Leftrightarrow \frac{M_e}{2} \frac{v_x^{\ 2}}{c^2} \tag{282}$$

The equation reduces to,

$$\frac{\mu_0 \left(e^-\right)^2}{8\pi r_e} \Leftrightarrow \frac{M_e}{2} \tag{283}$$

So, given,

- Permeability of free space $\mu_0 = 4\pi \times 10^{-7}\ H/m$
- Fundamental charge of an electron $e^- = 1.602177 \times 10^{-19} C$
- Classic electron radius $r_e = 2.817941 \times 10^{-15} m$
- Rest mass of an electron $M_e = 9.109390 \times 10^{-31} kg$

$$\frac{\left(4\pi \times 10^{-7}\ H/m\right)\left(1.602177 \times 10^{-19} C\right)^2}{8\pi\left(2.817941 \times 10^{-15} m\right)} \Leftrightarrow \frac{9.109390 \times 10^{-31} kg}{2} \tag{284}$$

This shows the fluctuating magnetic mass of a moving electron is identical to the *special relativistic* mass at any velocity v_x,

$$4.554693 \Leftrightarrow 4.554695 \tag{285}$$

Therefore, the fluctuating magnetic mass dM_B **is** the fluctuating mass dM_e of an electron,

$$dM_B = dM_e \tag{286}$$

So, the magnetic mass M_B and the mass M_e of the electron is,

$$M_B = M_e = \frac{\mu_0 \left(e^-\right)^2}{4\pi r_e} \tag{287}$$

And the fluctuating magnetic mass ΔM_B and the fluctuating mass ΔM_e is,

$$\Delta M_B = \Delta M_e = \frac{\mu_0 \left(e^-\right)^2}{8\pi r_e} \frac{v_x^{\ 2}}{c^2} = \frac{\mu_0 \left(e^-\right)^2}{4\pi r_e} \frac{v_x^{\ 2}}{2c^2} = M_e \frac{v_x^{\ 2}}{2c^2} \tag{288}$$

So, the velocity of an electron is,

$$v_x = 2c\sqrt{\frac{2\pi r_e \,\Delta M_e}{\mu_0 \left(e^-\right)^2}} = c\sqrt{\frac{2\,\Delta M_e}{M_e}} \tag{289}$$

Therefore, if the fluctuating mass is **positive**, then the velocity of the electron is *real*. However, if the fluctuating mass is **negative**, then the velocity is *imaginary*.

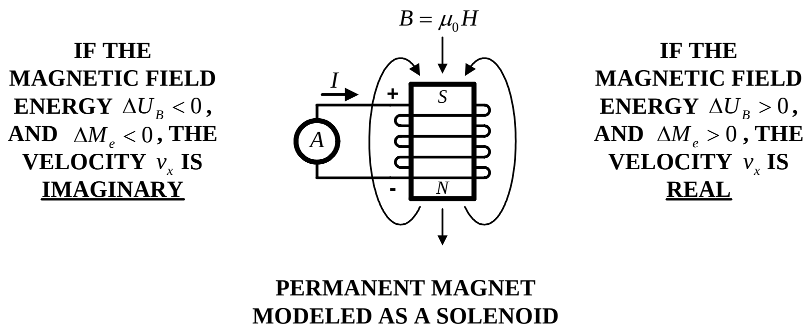


FIGURE 27. The magnetic fluctuating mass.

Now, the difference form of the inertial RED SHIFT (or BLUE SHIFT) of the magnetic mass $\Delta M_B / M_B$ and the mass $\Delta M_e / M_e$ of a particle moving at a velocity v_x is,

$$\frac{\Delta M_B}{M_B} = \frac{\Delta M_e}{M_e} = \frac{{v_x}^2}{2c^2} \tag{290}$$

A particle can move at a *real* (i.e., time-forward) velocity v_x, at an *imaginary* (i.e., time-future) velocity $j\,v_x$, or at a velocity that is a combination of the two. The *real* and *imaginary* components are rotated about the temporal axis and therefore, can be described as *complex* motion. The rotation is given as $0° \le \theta \le 90°$, where the *real* axis is $\theta = 0°$ and the *imaginary* time-future axis is $\theta = 90°$. The complex number uses the Euler's identity $e^{j\theta}$, which functions as a temporal rotation operator. The *complex* velocity v_x is,

$$v_x = v\,e^{j\theta} = v\cos\theta + j\,v\sin\theta \tag{291}$$

Given the rest mass of an electron M_e or the classic electron radius r_e, the difference forms of the *special relativistic* magnetic mass M_{ev} model of a particle moving at a *complex* velocity v_x, where $0° \le \theta \le 90°$ are,

$$M_{ev} = M_e \pm \Delta M_e = M_e\left(1 \pm \frac{{v_x}^2}{2c^2}\right) = M_e\, e^{\left(\frac{{v_x}^2}{2c^2}\right)} \tag{292}$$

$$M_{ev} = M_e \pm \Delta M_e = \frac{\mu_0\left(e^-\right)^2}{4\pi r_e}\left(1 \pm \frac{{v_x}^2}{2c^2}\right) = \frac{\mu_0\left(e^-\right)^2}{4\pi r_e}\, e^{\left(\frac{{v_x}^2}{2c^2}\right)} \tag{293}$$

Now, apply the new **Principle of Equivalence Theorem** where the fluctuating magnetic mass of a moving electron is equivalent to *natural relativistic* mass due to the Earth's gravity well,

$$\Delta M_e = \frac{\mu_0 \left(e^-\right)^2}{8\pi r_e} \frac{v_x^{\,2}}{c^2} = M_e \frac{v_x^{\,2}}{2c^2} = M_e \frac{\left(g_{y_1}\, y_1 - g_{y_0}\, y_0\right)}{c^2} = G\, M_E\, M_e \frac{\left(\frac{1}{y_1} - \frac{1}{y_0}\right)}{c^2} \tag{294}$$

$$v_x = \sqrt{2\left(g_{y_1}\, y_1 - g_{y_0}\, y_0\right)} = \sqrt{2G\, M_E \left(\frac{1}{y_1} - \frac{1}{y_0}\right)} \tag{295}$$

The position y_1 of an electron moving at a velocity v_x within Earth's gravity well g_Y where $0 < y_1 \le \infty$ or $-1 \le \frac{y_0\, v_x^{\,2}}{2G\, M_E}$ is,

$$y_1 = \frac{1}{g_{y_1}} \left(g_{y_0}\, y_0 + \frac{v_x^{\,2}}{2} \right) = \frac{1}{g_{y_1}} \left(g_{y_0}\, y_0 + \frac{\Delta M_e\, c^2}{M_e} \right) \tag{296}$$

$$y_1 = \frac{y_0}{1 + \frac{y_0\, v_x^{\,2}}{2G\, M_E}} = \frac{y_0}{1 + \frac{y_0\, c^2\, \Delta M_e}{G\, M_E\, M_e}} \tag{297}$$

The equivalent maximum *complex* velocity $v_{x\max}$ at $y_1 = \infty$ is,

$$v_{x\max} = \sqrt{-\frac{2G\, M_E}{y_0}} \tag{298}$$

Given the equivalent maximum *complex* velocity $v_{x\max}$, the minimum gravitational mass $M_{e\min}$ at $y_1 = \infty$ is,

$$M_{e\min} = M_e \left(1 + \frac{v_x^{\,2}}{2c^2} \right) = M_e \left(1 - \frac{G\, M_E}{y_0\, c^2} \right) = M_e\, e^{\left(-\frac{G\, M_E}{y_0\, c^2} \right)} \tag{299}$$

The equivalent maximum fluctuating gravitational mass of the electron ΔM_e at $y_1 = \infty$ is,

$$\Delta M_{e\max} = M_{e\min} - M_e = -\frac{G\, M_E\, M_e}{y_0\, c^2} \tag{300}$$

So, the difference form of the gravitational RED SHIFT (or BLUE SHIFT) of the magnetic mass $\Delta M_B / M_B$ and the mass $\Delta M_e / M_e$ of a particle displaced a distance Δy within a given gravity well g_Y is,

$$\frac{\Delta M_B}{M_B} = \frac{\Delta M_e}{M_e} = \frac{\left(g_{y_1}\, y_1 - g_{y_0}\, y_0\right)}{c^2} \tag{301}$$

Given the rest mass of an electron M_e or the classic electron radius r_e, the difference forms of the *natural relativistic* mass M_{ey_1} model of a particle displaced a distance Δy within a given gravity well g_Y are,

$$M_{ey_1} = M_e \pm \Delta M_e = M_e \left(1 \pm \frac{\left(g_{y_1}\, y_1 - g_{y_0}\, y_0 \right)}{c^2} \right) = M_e\, e^{\left(\frac{g_{y_1} y_1 - g_{y_0} y_0}{c^2} \right)} \tag{302}$$

$$M_{ey_1} = M_e \pm \Delta M_e = \frac{\mu_0 \left(e^- \right)^2}{4\pi\, r_e} \left(1 \pm \frac{\left(g_{y_1}\, y_1 - g_{y_0}\, y_0 \right)}{c^2} \right) = \frac{\mu_0 \left(e^- \right)^2}{4\pi\, r_e} e^{\left(\frac{g_{y_1} y_1 - g_{y_0} y_0}{c^2} \right)} \tag{303}$$

In summary, Gravitomagnetic Theory shows that a moving electron produces an increase in *relativistic* mass that extends from its' classic radius r_e to infinity, and couples to gravity. This motion can either have a velocity v_x or a *complex* (i.e., time-future) velocity $j\,v_x$. If the velocity is *complex*, then the electron will exhibit an antigravitational effect, and produce a *complex* (i.e., time-future) magnetic field $j\,B$. In addition, the total field energy U_B of a *complex* magnetic field $j\,B$ contained within a volume $\mathcal{V}$ is NEGATIVE.

Example 6. An electron e^- moving through a wire at a time-forward velocity v_x where $\theta = 0°$ produces a time-forward magnetic induction B at a distance r.

Given,

Direction of time is forward $\theta = 0°$
Velocity of electron e^- through a wire $v = 1.0 \times 10^{-2}\ m/\sec$
Permeability of free space $\mu_0 = 4\pi \times 10^{-7}\ H/m$
Fundamental charge of an electron $e^- = 1.602177 \times 10^{-19}\, C$
Rest mass of an electron $M_e = 9.109390 \times 10^{-31}\, kg$
Radius $r = 1.0\, m$
Gravitational constant $G = 6.67260 \times 10^{-11}\ N\, m^2 / kg^2$
Radius of surface of Earth $y_0 = 6.3781 \times 10^6\, m$
Mass of the Earth $M_E = 5.9787 \times 10^{24}\ kg$

The time-forward velocity v_x, where $\theta = 0°$ is,

$$v_x = v\, e^{j\theta} = \left(1.0 \times 10^{-2}\ m/\sec \right) e^{j0°} = 1.0 \times 10^{-2}\ m/\sec \tag{304}$$

The time-forward magnetic induction B at distance r is,

$$B = \frac{\mu_0\, e^-\, v_x}{4\pi\, r^2} = \frac{\left(4\pi \times 10^{-7}\ H/m \right)\left(1.602177 \times 10^{-19}\ C \right)\left(1.0 \times 10^{-2}\ m/\sec \right)}{4\pi \left(1.0\, m \right)^2} \tag{305}$$

$$B = 1.602177 \times 10^{-28}\, T \tag{306}$$

The POSITIVE fluctuating mass ΔM_e of the electron e^- is,

$$\Delta M_e = M_e \frac{v_x^{\,2}}{2c^2} = \left(9.1093897\times10^{-31}\,kg\right)\frac{\left(1.0\times10^{-2}\ m/\sec\right)^2}{2\left(2.99792458\times10^{8}\ m/\sec\right)^2} \tag{307}$$

$$\Delta M_e = 5.067782\times10^{-52}\,kg \tag{308}$$

Applying the new **Principle of Equivalence Theorem**,

$$y_1 = \frac{y_0}{1+\dfrac{y_0\,v_x^{\,2}}{2\,G\,M_E}} = \frac{\left(6.3781\times10^{6}\,m\right)}{1+\dfrac{\left(6.3781\times10^{6}\,m\right)\left(1.0\times10^{-2}\ m/\sec\right)^2}{2\left(6.67260\times10^{-11}\ N\,m^2/kg^2\right)\left(5.9787\times10^{24}\ kg\right)}} \tag{309}$$

$$y_1 = 6.37809999999490\times10^{6}\,m \tag{310}$$

So, the equivalent POSITIVE displacement Δy is gravitational within the Earth's gravity well is,

$$\Delta y = y_0 - y_1 = 5.0981\times10^{-6}\,m \tag{311}$$

Example 7. An electron e^- moving through a wire at a time-advanced velocity v_x where $0° < \theta < 90°$ produces a time-advanced magnetic induction B at a distance r.

Given,

- Direction of time is advanced $\theta = 45°$
- Velocity of electron e^- through a wire $v = 1.0\times10^{-2}\ m/\sec$
- Permeability of free space $\mu_0 = 4\pi\times10^{-7}\ H/m$
- Fundamental charge of an electron $e^- = 1.602177\times10^{-19}\,C$
- Rest mass of an electron $M_e = 9.109390\times10^{-31}\,kg$
- Radius $r = 1.0\,m$
- Gravitational constant $G = 6.67260\times10^{-11}\ N\,m^2/kg^2$
- Radius of surface of Earth $y_0 = 6.3781\times10^{6}\,m$
- Mass of the Earth $M_E = 5.9787\times10^{24}\ kg$

The time-advanced velocity v_x, where $\theta = 45°$ is,

$$v_x = v\,e^{j\theta} = \left(1.0\times10^{-2}\ m/\sec\right)e^{j45°} = 7.071068\times10^{-3} + 7.071068\,j\times10^{-3}\ m/\sec \tag{312}$$

The time-advanced magnetic induction B at distance r is,

$$B = \frac{\mu_0\,e^-\,v_x}{4\pi\,r^2} = \frac{\left(4\pi\times10^{-7}\ H/m\right)\left(1.602177\times10^{-19}\ C\right)\left(7.071068\times10^{-3} + 7.071068\,j\times10^{-3}\ m/\sec\right)}{4\pi\left(1.0\,m\right)^2} \tag{313}$$

$$B = 1.132910\times10^{-28} + 1.132910\,j\times10^{-28}\,T \tag{314}$$

The IMAGINARY fluctuating mass ΔM_e of the electron e^- is,

$$\Delta M_e = M_e \frac{{v_x}^2}{2c^2} = \left(9.1093897\times10^{-31}kg\right)\frac{\left(7.071068\times10^{-3}+7.071068\,j\times10^{-3}\ m/\sec\right)^2}{2\left(2.99792458\times10^{8}\ m/\sec\right)^2} \tag{315}$$

$$\Delta M_e = 5.067782\,j\times10^{-52}kg \tag{316}$$

Applying the new **Principle of Equivalence Theorem**,

$$y_1 = \frac{y_0}{1+\frac{y_0\,{v_x}^2}{2G\,M_E}} = \frac{\left(6.3781\times10^6 m\right)}{1+\frac{\left(6.3781\times10^6 m\right)\left(7.071068\times10^{-3}+7.071068\,j\times10^{-3}\ m/\sec\right)^2}{2\left(6.67260\times10^{-11}\ N\,m^2/kg^2\right)\left(5.9787\times10^{24}\ kg\right)}} \tag{317}$$

$$y_1 = 6.3781\times10^6 m - 5.0986\,j\times10^{-6} m \tag{318}$$

So, the equivalent IMAGINARY displacement Δy is shown to be non-gravitational within the Earth's gravity well is,

$$\Delta y = y_0 - y_1 = 5.0986\,j\times10^{-6} m \tag{319}$$

The maximum time-future velocity $v_{x\max}$ within the Earth's gravity well is,

$$v_{x\max} = \sqrt{-\frac{2G\,M_E}{y_0}} = \sqrt{-\frac{2\left(6.67260\times10^{-11}\ N\,m^2/kg^2\right)\left(5.9787\times10^{24}\ kg\right)}{\left(6.3781\times10^6 m\right)}} \tag{320}$$

$$v_{x\max} = 1.11846\,j\times10^4\ m/\sec \tag{321}$$

Example 8. An electron e^- moving through a wire at a time-future velocity v_x where $\theta = 90°$ produces a time-future magnetic induction B at a distance r.

Given,

- Direction of time is future $\theta = 90°$
- Velocity of electron e^- through a wire $v = 1.0\times10^{-2}\ m/\sec$
- Permeability of free space $\mu_0 = 4\pi\times10^{-7}\ H/m$
- Fundamental charge of an electron $e^- = 1.602177\times10^{-19} C$
- Rest mass of an electron $M_e = 9.109390\times10^{-31} kg$
- Radius $r = 1.0\,m$
- Gravitational constant $G = 6.67260\times10^{-11}\ N\,m^2/kg^2$
- Radius of surface of Earth $y_0 = 6.3781\times10^6 m$
- Mass of the Earth $M_E = 5.9787\times10^{24}\ kg$

The time-future velocity v_x, where $\theta = 90°$ is,

$$v_x = v\,e^{j\theta} = \left(1.0\times10^{-2}\ m/\sec\right)e^{j90°} = 1.0\,j\times10^{-2}\ m/\sec \tag{322}$$

The time-future magnetic induction B at distance r is,

$$B = \frac{\mu_0\, e^-\, v_x}{4\pi\, r^2} = \frac{\left(4\pi\times10^{-7}\ H/m\right)\left(1.602177\times10^{-19}\ C\right)\left(1.0\,j\times10^{-2}\ m/\sec\right)}{4\pi\left(1.0\,m\right)^2} \tag{323}$$

$$B = 1.602177\,j\times10^{-28}\,T \tag{324}$$

The NEGATIVE fluctuating mass ΔM_e of the electron e^- is,

$$\Delta M_e = M_e\,\frac{{v_x}^2}{2c^2} = \left(9.1093897\times10^{-31}\,kg\right)\frac{\left(1.0\,j\times10^{-2}\ m/\sec\right)^2}{2\left(2.99792458\times10^{8}\ m/\sec\right)^2} \tag{325}$$

$$\Delta M_e = -5.067782\times10^{-52}\,kg \tag{326}$$

Applying the new **Principle of Equivalence Theorem**,

$$y_1 = \frac{y_0}{1+\dfrac{y_0\,{v_x}^2}{2\,G\,M_E}} = \frac{\left(6.3781\times10^{6}\,m\right)}{1+\dfrac{\left(6.3781\times10^{6}\,m\right)\left(1.0\,j\times10^{-2}\ m/\sec\right)^2}{2\left(6.67260\times10^{-11}\ N\,m^2/kg^2\right)\left(5.9787\times10^{24}\ kg\right)}} \tag{327}$$

$$y_1 = 6.37800000000510\times10^{6}\,m \tag{328}$$

So, the equivalent NEGATIVE displacement Δy is antigravitational within the Earth's gravity well is,

$$\Delta y = y_0 - y_1 = -5.0981\times10^{-6}\,m \tag{329}$$

The maximum time-future velocity $v_{x\max}$ within the Earth's gravity well is,

$$v_{x\max} = \sqrt{-\frac{2\,G\,M_E}{y_0}} = \sqrt{-\frac{2\left(6.67260\times10^{-11}\ N\,m^2/kg^2\right)\left(5.9787\times10^{24}\ kg\right)}{\left(6.3781\times10^{6}\,m\right)}} \tag{330}$$

$$v_{x\max} = 1.11846\,j\times10^{4}\ m/\sec \gg v_x \tag{331}$$

THE RELATIVITY OF ORBITAL SPIN

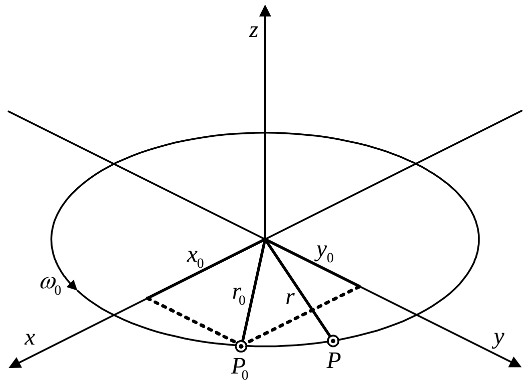


FIGURE 28. The definition of a rotating space-time interval.

A linear space-time interval is defined as,

$$ds_0^{\,2} + dr^2 = c^2 dt^2 \qquad (332)$$

Since $z_0 = 0$,

$$r_0^{\,2} = x_0^{\,2} + y_0^{\,2} + z_0^{\,2} = x_0^{\,2} + y_0^{\,2} \qquad (333)$$

$$dr^2 = dx^2 + dy^2 + dz^2 = dx^2 + dy^2 \qquad (334)$$

According to Fock (1964), a rotating space-time interval is defined as,

$$x = x_0 \cos \omega t + y_0 \sin \omega t \qquad (335)$$

$$y = -x_0 \sin \omega t + y_0 \cos \omega t \qquad (336)$$

$$ds_0^{\,2} = \left(c^2 - \omega_0^{\,2}\left(x_0^{\,2} + y_0^{\,2}\right)\right)dt^2 - 2\omega\left(y_0\, dx - x_0\, dy\right)dt - \left(dx^2 + dy^2 + dz^2\right) \qquad (337)$$

This reduces to,

$$ds_0^{\,2} = \left(c^2 - r_0^{\,2}\omega_0^{\,2}\right)dt^2 - 2\omega_0\left(y_0\, dx - x_0\, dy\right)dt - dr^2 \qquad (338)$$

COMPLEX AMPERIAN CURRENTS

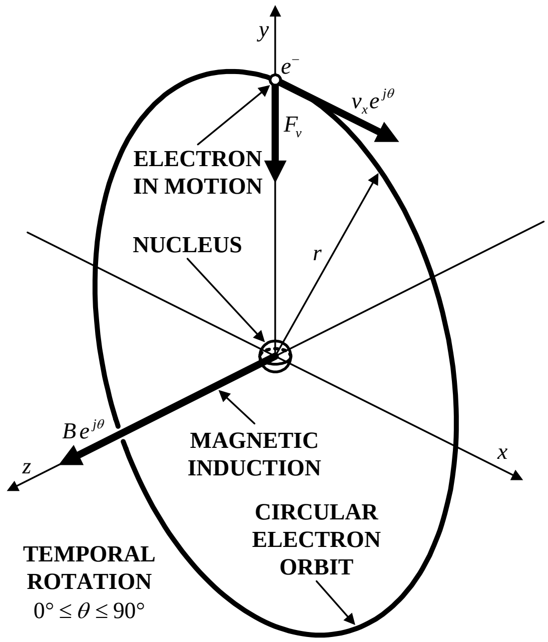


FIGURE 29. The complete *complex* Bohr model of the Hydrogen atom.

In the Bohr model of the Hydrogen atom, electrons move at relativistic speeds in discrete circular orbits around a nucleus. It has been determined the increase in relativistic mass of the electron is in the form of total magnetic field energy produced by the circulating electron. So, as a consequence of this motion, a magnetic induction B is produced at the center of the orbit. If an external magnetic field is applied to this induction, the velocity of the electron becomes complex by partially rotating into the imaginary axis. The velocity of the electron may increase or decrease as a function of the applied external field. If the field opposes the induction B, the *real* velocity will appear to decrease as it rotates into the imaginary axis. If the electron's velocity is fully rotated into the imaginary axis and therefore moving at a time-future velocity $j v_x$, a time-future magnetic field $j B$ will emerge from the center. The complete *complex* Bohr model includes the following characteristic equations shown below. These equations contain the *real* and *imaginary* components of a moving electron that is rotated about the temporal axis as *complex* motion. The rotation is given as $0° \le \theta \le 90°$, where the *real* axis is $\theta = 0°$ and the *imaginary* time-future axis is $\theta = 90°$. The *complex* number uses the Euler's identity $e^{j\theta}$, which functions as a temporal rotation operator.

So, given,

- Direction of time θ
- Frequency of orbit f
- Permeability of free space μ_0
- Fundamental charge of an electron e^-
- Classic 1st Bohr orbital radius r
- Rest mass of an electron M_e
- Speed of light c
- Gravitational constant G
- Mean radius of surface of Earth y_0
- Mass of the Earth M_E
- Mean radius of surface of Sun y_0
- Mass of the Sun M_{SUN}

The *complex* frequency of orbit f_0, where $0° \le \theta \le 90°$ is,

$$f_0 = f\, e^{j\theta} = f \cos\theta + j\, f \sin\theta \tag{339}$$

The *complex* Amperian Current i of the electron is,

$$i = e^{-} f_0 \tag{340}$$

The *complex* Magnetic Dipole Moment μ of the electron is,

$$\mu = \pi\, r^2\, i = \pi\, r^2\, e^{-}\; f_0 \tag{341}$$

The *complex* angular velocity ω_0 of the electron is,

$$\omega_0 = 2\,\pi\, f_0 \tag{342}$$

The *complex* velocity v_x of the electron is,

$$v_x = r\,\omega_0 = 2\,\pi\, r\; f_0 \tag{343}$$

The *complex* magnetic field B at the center axis of the orbit $z = 0$ is,

$$B = \frac{\mu_0\, r^2\, i}{2\left(r^2 + z^2\right)^{3/2}} = \frac{\mu_0\,\mu}{2\pi\left(r^2 + z^2\right)^{3/2}} = \frac{\mu_0\, r^2\, e^{-}\, f_0}{2\left(r^2 + z^2\right)^{3/2}} = \frac{\mu_0\, r^2\, e^{-}\,\omega_0}{4\pi\left(r^2 + z^2\right)^{3/2}} = \frac{\mu_0\, r\, e^{-}\, v_x}{4\pi\left(r^2 + z^2\right)^{3/2}} \tag{344}$$

$$B = \frac{\mu_0\, i}{2\, r} = \frac{\mu_0\,\mu}{2\,\pi\, r^3} = \frac{\mu_0\, e^{-}\, f_0}{2\, r} = \frac{\mu_0\, e^{-}\,\omega_0}{4\,\pi\, r} = \frac{\mu_0\, e^{-}\, v_x}{4\,\pi\, r^2} \tag{345}$$

The magnetic force F_v of the electron e^{-} directed upon the nucleus is,

$$F_v = e^{-}\, v_x \times B \tag{346}$$

$$F_v = \mu_0\,\pi\, i^2 = \frac{\mu_0\,\mu^2}{\pi\, r^4} = \mu_0\,\pi\left(e^{-}\right)^2\; f_0^{\;2} = \frac{\mu_0\left(e^{-}\right)^2\,\omega_0^{\;2}}{4\,\pi} = \frac{\mu_0\left(e^{-}\right)^2\, v_x^{\;2}}{4\,\pi\, r^2} \tag{347}$$

The direction of electron motion v_x is such that the magnetic force F_v is an attractive force between the electron and the nucleus.

The *special relativistic* mass M_{ev} of the circulating electron is,

$$M_{ev} = M_e \pm \Delta M_e = M_e\left(1 \pm \frac{v_x^{\;2}}{2\,c^2}\right) = M_e\, e^{\left(\frac{v_x^{\;2}}{2\,c^2}\right)} \tag{348}$$

The change in relativistic mass of the electron ΔM_e is in the form of the change in magnetic field energy ΔU_B produced by the circulating electron.

$$\Delta U_B = \Delta E_e = \Delta M_e\, c^2 \tag{349}$$

The difference form of the inertial RED SHIFT (or BLUE SHIFT) of the mass $\Delta M_e / M_e$ of an electron moving at a velocity v_x is,

$$\frac{\Delta M_e}{M_e} = \frac{{v_x}^2}{2\,c^2} = \frac{2\,\pi^2\, r^2\, i^2}{\left(e^-\right)^2 c^2} = \frac{2\,\mu^2}{\left(e^-\right)^2 c^2\, r^2} = \frac{2\,\pi^2\, r^2\, f_0^{\,2}}{c^2} = \frac{r^2\, {\omega_0}^2}{2\,c^2} \tag{350}$$

So, the *complex* velocity v_x of the electron is,

$$v_x = \frac{2\,r}{e^-}\sqrt{\frac{\pi\, F_v}{\mu_0}} = c\sqrt{\frac{2\,\Delta M_e}{M_e}} \tag{351}$$

The *complex* Amperian Current i of the electron is,

$$i = \sqrt{\frac{F_v}{\mu_0\,\pi}} = \frac{e^-\, c}{\pi\, r}\sqrt{\frac{\Delta M_e}{2\,M_e}} \tag{352}$$

The *complex* Magnetic Dipole Moment μ of the electron is,

$$\mu = r^2\sqrt{\frac{\pi\, F_v}{\mu_0}} = e^- c\, r\sqrt{\frac{\Delta M_e}{2\,M_e}} \tag{353}$$

The *complex* frequency of orbit f_0 of the electron is,

$$f_0 = \frac{1}{e^-}\sqrt{\frac{F_v}{\mu_0\,\pi}} = \frac{c}{\pi\, r}\sqrt{\frac{\Delta M_e}{2\,M_e}} \tag{354}$$

The *complex* angular velocity ω_0 of the electron is,

$$\omega_0 = \frac{2}{e^-}\sqrt{\frac{\pi\, F_v}{\mu_0}} = \frac{c}{r}\sqrt{\frac{2\,\Delta M_e}{M_e}} \tag{355}$$

So, the *complex* magnetic force F_v of the electron e^- directed upon the nucleus is,

$$F_v = \frac{\mu_0\left(e^-\right)^2 c^2}{2\,\pi\, r^2}\frac{\Delta M_e}{M_e} = \frac{\mu_0\left(e^-\right)^2}{2\,\pi\, r^2}\frac{\Delta E_e}{M_e} == \frac{\mu_0\left(e^-\right)^2}{2\,\pi\, r^2}\frac{\Delta U_B}{M_e} \tag{356}$$

And the difference form of the inertial RED SHIFT (or BLUE SHIFT) of the mass $\Delta M_e / M_e$ of an electron is,

$$\frac{\Delta M_e}{M_e} = \frac{2\,\pi\, r^2\, F_v}{\mu_0\left(e^-\right)^2 c^2} \tag{357}$$

Applying the new **Principle of Equivalence Theorem**,

$$\Delta M_e = M_e \frac{{v_x}^2}{2c^2} = M_e \frac{r^2 {\omega_0}^2}{2c^2} = M_e \frac{2\pi^2 r^2 {f_0}^2}{c^2} = M_e \frac{2\pi^2 r^2 i^2}{\left(e^-\right)^2 c^2} = M_e \frac{2\mu^2}{\left(e^-\right)^2 c^2 r^2} = M_e \frac{2\pi r^2 F_v}{\mu_0 \left(e^-\right)^2 c^2} \qquad (358)$$

$$\Delta M_e = M_e \frac{\left(g_{y_1} y_1 - g_{y_0} y_0\right)}{c^2} = G M_E M_e \frac{\left(\frac{1}{y_1} - \frac{1}{y_0}\right)}{c^2} \qquad (359)$$

So, the *complex* velocity v_x of the electron is,

$$v_x = \sqrt{2\left(g_{y_1} y_1 - g_{y_0} y_0\right)} = \sqrt{2 G M_E \left(\frac{1}{y_1} - \frac{1}{y_0}\right)} \qquad (360)$$

The *complex* angular velocity ω_0 of the electron is,

$$\omega_0 = \frac{1}{r}\sqrt{2\left(g_{y_1} y_1 - g_{y_0} y_0\right)} = \frac{1}{r}\sqrt{2 G M_E \left(\frac{1}{y_1} - \frac{1}{y_0}\right)} \qquad (361)$$

The *complex* frequency of orbit f_0 of the electron is,

$$f_0 = \frac{1}{\pi r}\sqrt{\frac{g_{y_1} y_1 - g_{y_0} y_0}{2}} = \frac{1}{\pi r}\sqrt{\frac{G M_E}{2}\left(\frac{1}{y_1} - \frac{1}{y_0}\right)} \qquad (362)$$

The *complex* Amperian Current i of the electron is,

$$i = \frac{e^-}{\pi r}\sqrt{\frac{g_{y_1} y_1 - g_{y_0} y_0}{2}} = \frac{e^-}{\pi r}\sqrt{\frac{G M_E}{2}\left(\frac{1}{y_1} - \frac{1}{y_0}\right)} \qquad (363)$$

The *complex* Magnetic Dipole Moment μ of the electron is,

$$\mu = e^- r\sqrt{\frac{g_{y_1} y_1 - g_{y_0} y_0}{2}} = e^- r\sqrt{\frac{G M_E}{2}\left(\frac{1}{y_1} - \frac{1}{y_0}\right)} \qquad (364)$$

The *complex* magnetic force F_v of the electron e^- directed upon the nucleus is,

$$F_v = \frac{\mu_0 \left(e^-\right)^2 \left(g_{y_1} y_1 - g_{y_0} y_0\right)}{2\pi r^2} = \frac{\mu_0 \left(e^-\right)^2 G M_E}{2\pi r^2}\left(\frac{1}{y_1} - \frac{1}{y_0}\right) \qquad (365)$$

The equivalent displacement to position y_1 of an electron moving at a velocity v_x within Earth's gravity well g_Y where $0 < y_1 \le \infty$ or $-1 \le \frac{y_0\, v_x{}^2}{2\,G\,M_E}$ is,

$$y_1 = \frac{1}{g_{y_1}}\left(g_{y_0}\; y_0 + \frac{v_x{}^2}{2} \right) = \frac{y_0}{1 + \frac{y_0\, v_x{}^2}{2\,G\,M_E}} \tag{366}$$

$$y_1 = \frac{1}{g_{y_1}}\left(g_{y_0}\; y_0 + \frac{r^2\, \omega_0{}^2}{2} \right) = \frac{y_0}{1 + \frac{y_0\, r^2\, \omega_0{}^2}{2\,G\,M_E}} \tag{367}$$

$$y_1 = \frac{1}{g_{y_1}}\left(g_{y_0}\; y_0 + 2\,\pi^2\, r^2\; f_0{}^2 \right) = \frac{y_0}{1 + \frac{2\,\pi^2\, y_0\, r^2\; f_0{}^2}{G\,M_E}} \tag{368}$$

$$y_1 = \frac{1}{g_{y_1}}\left(g_{y_0}\; y_0 + \frac{2\,\pi^2\, r^2\, i^2}{\left(e^-\right)^2} \right) = \frac{y_0}{1 + \frac{2\,\pi^2\, y_0\, r^2\, i^2}{\left(e^-\right)^2\, G\,M_E}} \tag{369}$$

$$y_1 = \frac{1}{g_{y_1}}\left(g_{y_0}\; y_0 + \frac{2\,\mu^2}{\left(e^-\right)^2\, r^2} \right) = \frac{y_0}{1 + \frac{2\, y_0\, \mu^2}{\left(e^-\right)^2\, r^2 G\,M_E}} \tag{370}$$

$$y_1 = \frac{1}{g_{y_1}}\left(g_{y_0}\; y_0 + \frac{2\,\pi\, r^2\, F_v}{\mu_0 \left(e^-\right)^2} \right) = \frac{y_0}{1 + \frac{2\,\pi\, r^2 y_0\, F_v}{\mu_0 \left(e^-\right)^2\, G\,M_E}} \tag{371}$$

The difference form of the gravitational RED SHIFT (or BLUE SHIFT) of the mass $\Delta M_e / M_e$ of a particle displaced a distance Δy within Earth's gravity well g_Y is,

$$\frac{\Delta M_e}{M_e} = \frac{\left(g_{y_1}\; y_1 - g_{y_0}\; y_0 \right)}{c^2} = \frac{G\,M_E}{c^2}\left(\frac{1}{y_1} - \frac{1}{y_0} \right) \tag{372}$$

Given the rest mass of an electron M_e, the difference forms of the *natural relativistic* mass M_{ey_1} model of a particle displaced a distance Δy within Earth's gravity well g_Y are,

$$M_{ey_1} = M_e \pm \Delta M_e = M_e \left(1 \pm \frac{\left(g_{y_1}\; y_1 - g_{y_0}\; y_0 \right)}{c^2} \right) = M_e\, e^{\left(\frac{g_{y_1}\; y_1 - g_{y_0} y_0}{c^2} \right)} \tag{373}$$

$$M_{ey_1} = M_e \pm \Delta M_e = M_e\left(1 \pm \frac{G\,M_E}{c^2}\left(\frac{1}{y_1} - \frac{1}{y_0}\right)\right) = M_e\, e^{\left(\frac{G\,M_E}{c^2}\left(\frac{1}{y_1} - \frac{1}{y_0}\right)\right)} \tag{374}$$

In summary, if the *real* component of the magnetic field is cancelled, or $B = 0$, due to an externally applied magnetic field B_{EXT}, the velocity of the circulating electron is *complex*, or $j\,v_x$ and a *complex* magnetic field $j\,B$ emerges. This *complex* magnetic field is believed to be present in the Aharonov-Bohm Experiment, which affected the flow of electrons. The *complex* Amperian Current uses the temporal rotation operator or Euler's identity $e^{j\theta}$, where $0° \le \theta \le 90°$. As the motion of an electron rotates from *real* to *imaginary*, or $\theta \to 90°$, the electrons *special relativistic* mass ΔM_e in the form of magnetic field energy ΔU_B decreases. In addition, the electrons rest mass M_e as well as its charge is invariant during acceleration or deceleration.

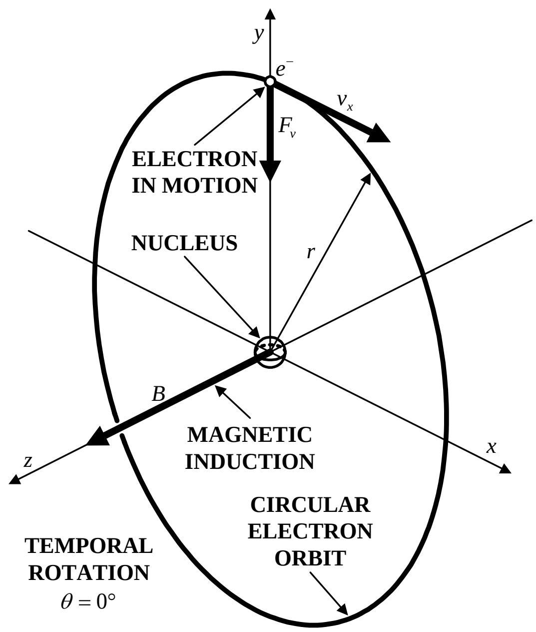


FIGURE 30. The time-forward Bohr model of the Hydrogen atom.

Example 9. In the time-forward Bohr model of the Hydrogen atom the electron e^- circulates around the nucleus at a *relativistic* velocity v_x as shown above. This creates a magnetic induction B emerging from the center of the nucleus.

So, given,

Direction of time is forward $\theta = 0°$

Frequency of orbit $f = 6.8 \times 10^{15}\,Hz$

Permeability of free space $\mu_0 = 4\pi \times 10^{-7}\; H/m$

Fundamental charge of an electron $e^- = 1.602177 \times 10^{-19}\,C$

Classic 1st Bohr orbital radius $r = 5.291773 \times 10^{-11}\,m$

Rest mass of an electron $M_e = 9.1093897 \times 10^{-31}\,kg$

Speed of light $c = 2.99792458 \times 10^{8}\; m/\sec$

Gravitational constant $G = 6.67260 \times 10^{-11}\; N\,m^2/kg^2$

Mean radius of surface of Earth $y_0 = 6.3781 \times 10^{6}\,m$

Mass of the Earth $M_E = 5.9787 \times 10^{24}\, kg$

The time-forward frequency of orbit f_0, where $\theta = 0°$ is,

$$f_0 = f\, e^{j\theta} = \left(6.8 \times 10^{15}\, Hz\right) e^{j0°} = 6.8 \times 10^{15}\, Hz \tag{375}$$

The time-forward Amperian Current i is,

$$i = e^{-} f_0 = \left(1.602177 \times 10^{-19}\, C\right)\left(6.8 \times 10^{15}\, Hz\right) = 1.089481 \times 10^{-3}\, Amps \tag{376}$$

The time-forward magnetic field B at the center axis of the orbit $z = 0$ is,

$$B = \mu_0 \frac{r^2\, i}{2\left(r^2 + z^2\right)^{3/2}} = \mu_0 \frac{r^2\, e^{-} f_0}{2\left(r^2 + z^2\right)^{3/2}} \tag{377}$$

$$B = \mu_0 \frac{i}{2\, r} = \left(4\pi \times 10^{-7}\, H/m\right) \frac{\left(1.089481 \times 10^{-3}\, Amps\right)}{2\left(5.291773 \times 10^{-11}\, m\right)} = 12.935946\, T \tag{378}$$

The time-forward angular velocity ω_0 of the electron e^{-} is,

$$\omega_0 = 2\,\pi\, f_0 = 2\,\pi \left(6.8 \times 10^{15}\, Hz\right) = 4.272566 \times 10^{16}\, Hz \tag{379}$$

The time-forward velocity v_x of the electron e^{-} is,

$$v_x = r\, \omega_0 = \left(5.291773 \times 10^{-11}\, m\right)\left(4.272566 \times 10^{16}\, Hz\right) = 2.260945 \times 10^{6}\, m/\sec \tag{380}$$

The time-forward magnetic force F_v of the electron e^{-} directed upon the nucleus is,

$$F_v = e^{-} v_x\, B = \left(1.602177 \times 10^{-19}\, C\right)\left(2.260945 \times 10^{6}\, m/\sec\right)\left(12.935946\, T\right) \tag{381}$$

$$F_v = 4.685962 \times 10^{-12}\, N \tag{382}$$

The direction of electron motion v_x is such that the magnetic force F_v is always an attractive force between the electron and the nucleus.

The POSITIVE fluctuating mass ΔM_e of the electron e^{-} is,

$$\Delta M_e = M_e \frac{{v_x}^2}{2\, c^2} = \left(9.1093897 \times 10^{-31}\, kg\right) \frac{\left(2.260945 \times 10^{6}\, m/\sec\right)^2}{2\left(2.99792458 \times 10^{8}\, m/\sec\right)^2} = 2.590585 \times 10^{-35}\, kg \tag{383}$$

The increased *special relativistic* mass M_v of the electron e^{-} is,

$$M_v = M_e + \Delta M_e = \left(9.1093897 \times 10^{-31}\, kg\right) + \left(2.590585 \times 10^{-35}\, kg\right) = 9.109649 \times 10^{-31}\, kg \tag{384}$$

Applying the new **Principle of Equivalence Theorem**,

$$y_1 = \frac{y_0}{1+\frac{y_0\, v_x{}^2}{2\,G\,M_E}} = \frac{\left(6.3781\times10^6\,m\right)}{1+\frac{\left(6.3781\times10^6\,m\right)\left(2.260945\times10^6\,m/\sec\right)^2}{2\left(6.67260\times10^{-11}\,N\,m^2/kg^2\right)\left(5.9787\times10^{24}\,kg\right)}} \tag{385}$$

$$y_1 = 156.0784\,m \tag{386}$$

So, the equivalent POSITIVE displacement Δy is gravitational within the Earth's gravity well is,

$$\Delta y = y_0 - y_1 = 6.377944\times10^6\,m \tag{387}$$

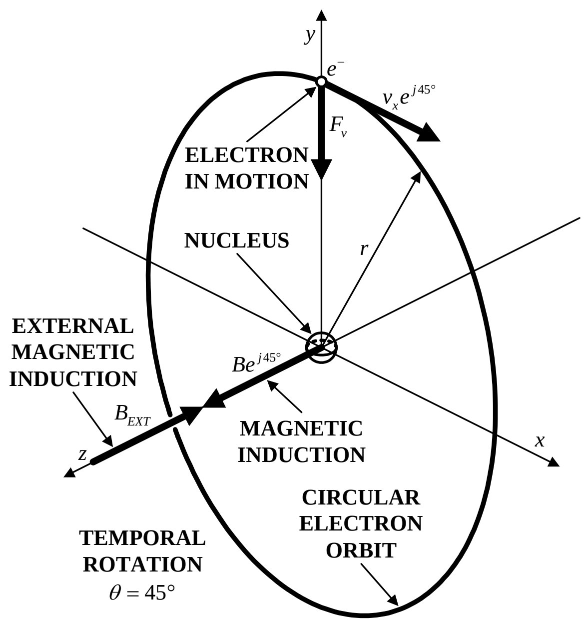


FIGURE 31. The time-advanced Bohr model of the Hydrogen atom.

Example 10. In the time-advanced Bohr model of the Hydrogen atom, the *real* magnetic field created by an electron circulating at *relativistic* speeds is partially cancelled by an externally applied magnetic field B_{EXT}. The electron reacts by rotating its velocity into the imaginary axis as shown above. As a consequence of this *complex* velocity $v\,e^{j45°}$, a *complex* magnetic field $B\,e^{j45°}$ emerges.

So, given,

Direction of time $\theta = 45°$

Frequency of orbit $f = 6.8\times10^{15}\,Hz$

Permeability of free space $\mu_0 = 4\pi\times10^{-7}\,H/m$

Fundamental charge of an electron $e^- = 1.602177\times10^{-19}\,C$

Classic 1st Bohr orbital radius $r = 5.291773\times10^{-11}\,m$

Rest mass of an electron $M_e = 9.1093897\times10^{-31}\,kg$

Speed of light $c = 2.99792458\times10^8\,m/\sec$

Gravitational constant $G = 6.67260\times10^{-11}\,N\,m^2/kg^2$

Mean radius of surface of Earth $y_0 = 6.3781 \times 10^6 \, m$
Mass of the Earth $M_E = 5.9787 \times 10^{24} \, kg$
Mean radius of surface of Sun $y_0 = 6.96 \times 10^8 \, m$
Mass of the Sun $M_{SUN} = 1.98892 \times 10^{30} \, kg$

The time-advanced frequency of orbit f_0, where $\theta = 45°$ is,

$$f_0 = f\, e^{j\theta} = \left(6.8 \times 10^{15} \, Hz\right) e^{j45°} = 4.808326 \times 10^{15} + 4.808326\, j \times 10^{15} \, Hz \tag{388}$$

The time-advanced Amperian Current i is,

$$i = e^- f_0 = \left(1.602177 \times 10^{-19} \, C\right)\left(4.808326 \times 10^{15} + 4.808326\, j \times 10^{15} \, Hz\right) \tag{389}$$

$$i = 7.703791 \times 10^{-4} + 7.703791\, j \times 10^{-4} \, Amps \tag{390}$$

The time-advanced magnetic field B at the center axis of the orbit $z = 0\, m$ is,

$$B = \mu_0 \frac{r^2\, i}{2\left(r^2 + z^2\right)^{3/2}} = \mu_0 \frac{r^2\, e^-\, f_0}{2\left(r^2 + z^2\right)^{3/2}} \tag{391}$$

$$B = \mu_0 \frac{i}{2\, r} = \left(4\pi \times 10^{-7} \, H/m\right) \frac{\left(7.703791 \times 10^{-4} + 7.703791\, j \times 10^{-4} \, Amps\right)}{2\left(5.291773 \times 10^{-11} \, m\right)} \tag{392}$$

$$B = 9.147095 + 9.147095\, j\, T \tag{393}$$

The time-advanced angular velocity ω_0 of the electron e^- is,

$$\omega_0 = 2\,\pi\, f_0 = 2\,\pi \left(4.808326 \times 10^{15} + 4.808326\, j \times 10^{15} \, Hz\right) \tag{394}$$

$$\omega_0 = 3.02116 \times 10^{16} + 3.02116\, j \times 10^{16} \, Hz \tag{395}$$

The time-advanced velocity v_x of the electron e^- is,

$$v_x = r\, \omega_0 = \left(5.291773 \times 10^{-11} \, m\right)\left(3.02116 \times 10^{16} + 3.02116\, j \times 10^{16} \, Hz\right) \tag{396}$$

$$v_x = 1.598729 \times 10^6 + 1.598729\, j \times 10^6 \, m/\sec \tag{397}$$

The maximum time-future velocity $v_{x\max}$ within the Earth's gravity well is,

$$v_{x\max} = \sqrt{-\frac{2\, G\, M_E}{y_0}} = \sqrt{-\frac{2\left(6.67260 \times 10^{-11} \, N\, m^2/kg^2\right)\left(5.9787 \times 10^{24} \, kg\right)}{\left(6.3781 \times 10^6 \, m\right)}} \tag{398}$$

$$v_{x\max} = 1.11846\, j \times 10^4 \, m/\sec \ll v_x \tag{399}$$

The time-advanced velocity v_x of the electron e^- far exceeds coupling to Earth's gravity well!

The maximum time-future velocity $v_{x\max}$ within the Sun's gravity well is,

$$v_{x\max} = \sqrt{-\frac{2\,G\,M_{SUN}}{y_0}} = \sqrt{-\frac{2\left(6.67260\times10^{-11}\,N\,m^2/kg^2\right)\left(1.98892\times10^{30}\,kg\right)}{\left(6.96\text{ x }10^8\text{ m}\right)}} \tag{400}$$

$$v_{x\max} = 6.17542\,j\times10^5\,m/\sec \ll v_x \tag{401}$$

The time-advanced velocity v_x of the electron e^- far exceeds coupling to Sun's gravity well!

The time-advanced magnetic force F_v of the electron e^- directed upon the nucleus is,

$$F_v = e^-\,v_x\,B \tag{402}$$

$$F_v = \left(1.602177\times10^{-19}\,C\right)\left(1.598729\times10^6 + 1.598729\,j\times10^6\,m/\sec\right)\left(9.147095 + 9.147095\,j\,T\right)$$

$$F_v = 4.685962\,j\times10^{-12}\,N \tag{403}$$

The direction of electron motion v_x is time-advanced or rotated into the future such that the magnetic force F_v is always an attractive force between the electron and the nucleus.

The IMAGINARY fluctuating mass ΔM_e of the electron e^- is,

$$\Delta M_e = M_e\,\frac{{v_x}^2}{2\,c^2} = \left(9.1093897\times10^{-31}\,kg\right)\frac{\left(1.598729\times10^6 + 1.598729\,j\times10^6\,m/\sec\right)^2}{2\left(2.99792458\times10^8\,m/\sec\right)^2} \tag{404}$$

$$\Delta M_e = 2.590585\,j\times10^{-35}\,kg \tag{405}$$

The *special relativistic* mass M_v of the electron e^- is,

$$M_v = M_e + \Delta M_e = \left(9.1093897\times10^{-31}\,kg\right) + \left(2.590585\,j\times10^{-35}\,kg\right) \tag{406}$$

$$M_v = 9.1093897\times10^{-31} + 2.590585\,j\times10^{-35}\,kg \tag{407}$$

Applying the new **Principle of Equivalence Theorem**,

$$y_1 = \frac{y_0}{1+\dfrac{y_0\,{v_x}^2}{2\,G\,M_E}} = \frac{\left(6.3781\times10^6\,m\right)}{1+\dfrac{\left(6.3781\times10^6\,m\right)\left(1.598729\times10^6 + 1.598729\,j\times10^6\,m/\sec\right)^2}{2\left(6.67260\times10^{-11}\,N\,m^2/kg^2\right)\left(5.9787\times10^{24}\,kg\right)}} \tag{408}$$

$$y_1 = 3.819579\times10^{-3} - 156.0822\,j\,m \tag{409}$$

So, the equivalent IMAGINARY displacement Δy is shown to be non-gravitational within the Earth's gravity well or,

$$\Delta y = y_0 - y_1 = 6.3781\times10^{-6} + 156.0822m \tag{410}$$

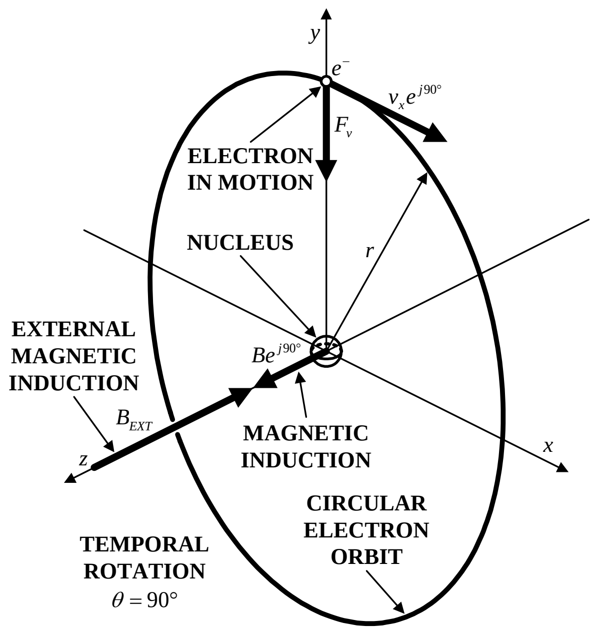


FIGURE 32. The time-future Bohr model of the Hydrogen atom.

Example 11. In the time-future Bohr model of the Hydrogen atom, the *real* magnetic field created by an electron circulating at *relativistic* speeds is being cancelled by an externally applied magnetic field B_{EXT}. The electron reacts by rotating its velocity into the imaginary axis as shown above. As a consequence of this *complex* velocity $j v_x$, a *complex* magnetic field $j B$ emerges.

So, given,

- Direction of time $\theta = 90°$
- Frequency of orbit $f = 6.8\times10^{15} Hz$
- Permeability of free space $\mu_0 = 4\pi\times10^{-7}\ H/m$
- Fundamental charge of an electron $e^- = 1.602177\times10^{-19} C$
- Classic 1st Bohr orbital radius $r = 5.291773\times10^{-11} m$
- Rest mass of an electron $M_e = 9.1093897\times10^{-31} kg$
- Speed of light $c = 2.99792458\times10^8\ m/\sec$
- Gravitational constant $G = 6.67260\times10^{-11}\ N\,m^2/kg^2$
- Mean radius of surface of Earth $y_0 = 6.3781\times10^6 m$
- Mass of the Earth $M_E = 5.9787\times10^{24}\ kg$
- Mean radius of surface of Sun $y_0 = 6.96\times10^8 m$
- Mass of the Sun $M_{SUN} = 1.98892\times10^{30}\ kg$

The time-future frequency of orbit f_0, where $\theta = 90°$ is,

$$f_0 = f\, e^{j\theta} = \left(6.8\times10^{15}\,Hz\right)e^{j90^\circ} = 6.8\,j\times10^{15}\,Hz \tag{411}$$

The time-future Amperian Current i is,

$$i = e^- f_0 = \left(1.602177\times10^{-19}\,C\right)\left(6.8\,j\times10^{15}\,Hz\right) = 1.089481\,j\times10^{-3}\,Amps \tag{412}$$

The time-future magnetic field B at the center axis of the orbit $z = 0\,m$ is,

$$B = \mu_0 \frac{r^2\, i}{2\left(r^2+z^2\right)^{3/2}} = \mu_0 \frac{r^2\, e^-\, f_0}{2\left(r^2+z^2\right)^{3/2}} \tag{413}$$

$$B = \mu_0 \frac{i}{2\,r} = \left(4\pi\times10^{-7}\,H/m\right)\frac{\left(1.089481\,j\times10^{-3}\,Amps\right)}{2\left(5.291773\times10^{-11}\,m\right)} = 12.935946\,j\,T \tag{414}$$

The time-future angular velocity ω_0 of the electron e^- is,

$$\omega_0 = 2\,\pi\, f_0 = 2\,\pi\left(6.8\,j\times10^{15}\,Hz\right) = 4.272566\,j\times10^{16}\,Hz \tag{415}$$

The time-future velocity v_x of the electron e^- is,

$$v_x = r\,\omega_0 = \left(5.291773\times10^{-11}\,m\right)\left(4.272566\,j\times10^{16}\,Hz\right) = 2.260945\,j\times10^{6}\,m/\sec \tag{416}$$

The maximum time-future velocity $v_{x\max}$ within the Earth's gravity well is,

$$v_{x\max} = \sqrt{-\frac{2\,G\,M_E}{y_0}} = \sqrt{-\frac{2\left(6.67260\times10^{-11}\,N\,m^2/kg^2\right)\left(5.9787\times10^{24}\,kg\right)}{\left(6.3781\times10^{6}\,m\right)}} \tag{417}$$

$$v_{x\max} = 1.11846\,j\times10^{4}\,m/\sec \ll v_x \tag{418}$$

The time-future velocity v_x of the electron e^- far exceeds coupling to Earth's gravity well.

The maximum time-future velocity $v_{x\max}$ within the Sun's gravity well is,

$$v_{x\max} = \sqrt{-\frac{2\,G\,M_{SUN}}{y_0}} = \sqrt{-\frac{2\left(6.67260\times10^{-11}\,N\,m^2/kg^2\right)\left(1.98892\times10^{30}\,kg\right)}{\left(6.96\text{ x }10^{8}\text{ m}\right)}} \tag{419}$$

$$v_{x\max} = 6.17542\,j\times10^{5}\,m/\sec \ll v_x \tag{420}$$

The time-advanced velocity v_x of the electron e^- far exceeds coupling to Sun's gravity well!

The magnetic force F_v of the electron e^- directed upon the nucleus is,

$$F_v = e^- v_x B = \left(1.602177\times10^{-19} C\right)\left(2.260945\, j\times10^{6}\, m/\sec\right)\left(12.935946\, j\, T\right) \tag{421}$$

$$F_v = -4.685962\times10^{-12} N \tag{422}$$

The direction of electron motion v_x is time-future or rotated into the future such that the magnetic force F_v is always an attractive force between the electron and the nucleus.

The NEGATIVE fluctuating mass ΔM_e of the electron e^- is,

$$\Delta M_e = M_e \frac{v_x^{\;2}}{2c^2} = \left(9.1093897\times10^{-31} kg\right)\frac{\left(2.260945\, j\times10^{6}\, m/\sec\right)^2}{2\left(2.99792458\times10^{8}\, m/\sec\right)^2} = -2.590585\times10^{-35} kg \tag{423}$$

The decreased *special relativistic* mass M_v of the electron e^- is,

$$M_v = M_e + \Delta M_e = \left(9.1093897\times10^{-31} kg\right) + \left(-2.590585\times10^{-35} kg\right) = 9.109131\times10^{-31} kg \tag{424}$$

Applying the new **Principle of Equivalence Theorem**,

$$y_1 = \frac{y_0}{1+\dfrac{y_0\, v_x^{\;2}}{2\,G\,M_E}} = \frac{\left(6.3781\times10^{6} m\right)}{1+\dfrac{\left(6.3781\times10^{6} m\right)\left(2.260945\, j\times10^{6}\, m/\sec\right)^2}{2\left(6.67260\times10^{-11}\, N\, m^2/kg^2\right)\left(5.9787\times10^{24}\, kg\right)}} \tag{425}$$

$$y_1 = -156.0860\, m \tag{426}$$

So, the equivalent NEGATIVE displacement Δy is antigravitational within the Earth's gravity well is,

$$\Delta y = y_0 - y_1 = 6.378256\times10^{6} m \tag{427}$$

COMPLEX ELECTRON DRIFT VELOCITY

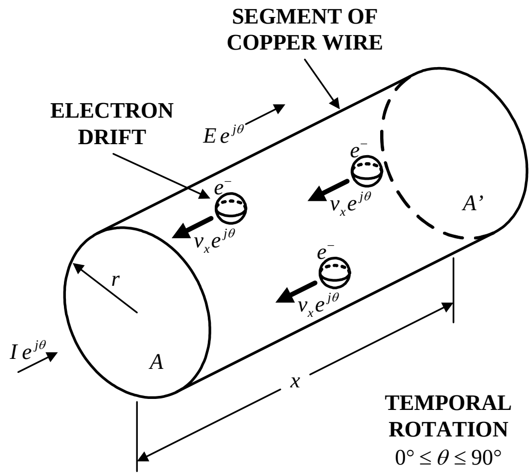


FIGURE 33. The complete *complex* electron drift velocity model in a copper wire.

If a copper wire is connected to a battery, an electric field E will be set up at every point within the wire. This field E will act on electrons and will give them a resultant motion. An electric current I is established if a net charge q_{Cu} passes through any cross sectional area A of the conductor in time t. The electric field that acts on the electrons doesn't produce a net acceleration because the electrons keep colliding with the atoms that make up the conductor. The electrons, therefore, move at an average drift velocity v_x. If the electron drift velocity $j\,v_x$ is time-future, the associated electric field $j\,E$ and magnetic field $j\,B$ are also time-future. The complete *complex* electron drift velocity model includes the following characteristic equations shown below. These equations contain the *real* and *imaginary* components of a moving electron that is rotated about the temporal axis as a complex particle. The rotation is given as $0° \le \theta \le 90°$, where the *real* axis is $\theta = 0°$ and the *imaginary* time-future axis is $\theta = 90°$. The *complex* number uses the Euler's identity $e^{j\theta}$, which functions as a temporal rotation operator.

So, given,

- Direction of time θ
- Current flowing through a conductor I
- Fundamental charge of an electron e^-
- Radius of copper wire r
- Density of conductor material $(20°C)$ D_{atom}
- Number of conduction electrons per atom of conductor k_{atom}
- Avogadro's Number N_0
- Atomic weight of conductor material W_{atom}
- Segment length of conductor x
- Speed of light c
- Rest mass of an electron M_e
- Radius of surface of Earth y_0
- Gravitational constant G
- Mass of the Earth M_E
- Resistivity of conductor material $(20°C)$ ρ_{atom}

The *complex* current I_0 flowing through a conductor, where $0° \le \theta \le 90°$ is,

$$I_0 = I\,e^{j\theta} = I\cos\theta + j\,I\sin\theta \tag{428}$$

The *complex* current density J_0 is,

$$J_0 = \frac{I_0}{A} = \frac{I_0}{\pi\,r^2} \tag{429}$$

The volume $\mathcal{V}$ of a segment of a conductor is,

$$\mathcal{V} = A\,x = \pi\,r^2 x \tag{430}$$

The quantity of conduction electrons n_{atom} in a volume of conductor is,

$$n_{atom} = \frac{D_{atom}\,N_0\,k_{atom}}{W_{atom}} \tag{431}$$

The net charge q_{atom} in a volume of a conductor is,

$$q_{atom} = n_{atom}\,\mathcal{V}\,e^- \tag{432}$$

The *complex* velocity v_x is,

$$v_x = \frac{x}{t} \tag{433}$$

The *complex* current I_0 flowing through a conductor is,

$$I_0 = \frac{q_{atom}}{t} = \frac{n_{atom}\,\mathcal{V}\,e^-}{x/v_x} = \pi\,r^2 n_{atom}\,e^-\,v_x \tag{434}$$

So, the *complex* drift velocity v_x of an electron moving through a conductor is,

$$v_x = \frac{I_0}{\pi\,r^2 n_{atom}\,e^-} = \frac{J_0}{n_{atom}\,e^-} \tag{435}$$

The fluctuating mass ΔM_e of the electron e^- is,

$$\Delta M_e = M_e\,\frac{{v_x}^2}{2\,c^2} \tag{436}$$

Applying the new **Principle of Equivalence Theorem**,

$$\Delta M_e = M_e\,\frac{{v_x}^2}{2\,c^2} = M_e\,\frac{\left(g_{y_1}\,y_1 - g_{y_0}\,y_0\right)}{c^2} = G\,M_E\,M_e\,\frac{\left(\frac{1}{y_1} - \frac{1}{y_0}\right)}{c^2} \tag{437}$$

$$\frac{{v_x}^2}{2} = g_{y_1}\, y_1 - g_{y_0}\, y_0 = G\, M_E \left(\frac{1}{y_1} - \frac{1}{y_0} \right) \tag{438}$$

$$v_x = \sqrt{2\left(g_{y_1}\, y_1 - g_{y_0}\, y_0\right)} = \sqrt{2\, G\, M_E \left(\frac{1}{y_1} - \frac{1}{y_0} \right)} \tag{439}$$

The equivalent displacement to position y_1 of an electron moving at a velocity v_x within Earth's gravity well g_Y where $0 < y_1 \le \infty$ or $-1 \le \frac{y_0\, {v_x}^2}{2\, G\, M_E}$ is,

$$y_1 = \frac{1}{g_{y_1}} \left(g_{y_0}\, y_0 + \frac{{v_x}^2}{2} \right) \tag{440}$$

$$y_1 = \frac{y_0}{1 + \frac{y_0\, {v_x}^2}{2\, G\, M_E}} \tag{441}$$

The resistivity ρ_{Atom} of a conductor is given as,

$$\rho_{Atom} = \frac{E}{J} = \frac{V/x}{I/A} = \frac{V/x}{I/\pi\, r^2} \tag{442}$$

The resistance R of a segment of conductor is,

$$R = \frac{V}{I} = \rho_{Atom}\, \frac{x}{A} = \rho_{Atom}\, \frac{x}{\pi\, r^2} \tag{443}$$

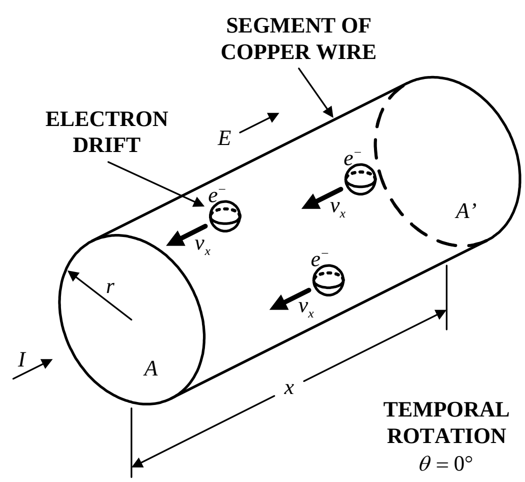


FIGURE 34. The time-forward electron drift velocity in a copper wire.

Example 12. A time-forward electric current I is established if a net charge q_{Cu} passes through any cross sectional area A of the conductor in time-forward t. The electrons move at an average time-forward drift velocity v_x.

So, given,

Direction of time $\theta = 0°$
Current through copper wire $I = 10.0\,Amps$
Fundamental charge of an electron $e^- = 1.602177\times10^{-19}\,C$
Radius of $10AWG$ copper wire $r = 1.294\times10^{-3}\,m$
Density of copper conductor $(20°C)$ $D_{Cu} = 8.92\times10^{6}\,gm/m^3$
Number of conduction electrons per atom of copper $k_{Cu} = 1\,electron/atom$
Avogadro's Number $N_0 = 6.0221367\times10^{23}\,atoms/mole$
Atomic weight of copper conductor $W_{Cu} = 63.546\,gm/mole$
Segment length $x = 1m$
Speed of light $c = 2.99792458\times10^{8}\,m/\sec$
Rest mass of an electron $M_e = 9.1093897\times10^{-31}\,kg$
Gravitational constant $G = 6.67260\times10^{-11}\,N\,m^2/kg^2$
Mean Radius of surface of Earth $y_0 = 6.3781\times10^{6}\,m$
Mass of the Earth $M_E = 5.9787\times10^{24}\,kg$
Resistivity of copper conductor $(20°C)$ $\rho_{Cu} = 1.68\times10^{-8}\,\Omega\,m$

The time-forward current I_0 flowing through a conductor, where $\theta = 0°$ is,

$$I_0 = I\,e^{j\theta} = (10.0\,Amps)\,e^{j0°} = 10.0\,Amps \tag{444}$$

The time-forward current density J_0 is,

$$J_0 = \frac{I_0}{A} = \frac{I_0}{\pi r^2} = \frac{(10.0\,Amps)}{\pi\left(1.294\times10^{-3}\,m\right)^2} = 1.901\times10^{6}\,Amps/m^2 \tag{445}$$

The volume $\mathcal{V}$ of a segment of copper wire is,

$$\mathcal{V} = A\,x = \pi r^2 x = \pi\left(1.294\times10^{-3}\,m\right)^2(1m) = 5.261\times10^{-6}\,m^3 \tag{446}$$

The quantity of conduction electrons n_{Cu} in a volume of copper wire is,

$$n_{Cu} = \frac{D_{Cu}\,N_0\,k_{Cu}}{W_{Cu}} \tag{447}$$

$$n_{Cu} = \frac{\left(8.92\times10^{6}\,gm/m^3\right)\left(6.0221367\times10^{23}\,atoms/mole\right)(1electron/atom)}{(63.546\,gm/mole)} \tag{448}$$

$$n_{Cu} = 8.4533\times10^{28}\,electrons/m^3 \tag{449}$$

The net charge q_{Cu} in a volume of copper wire is,

$$q_{Cu} = n_{Cu}\,\mathcal{V}\,e^- = \left(8.4533\times10^{28}\ electrons/m^3\right)\left(5.261\times10^{-6}\,m^3\right)\left(1.60217733\times10^{-19}\,C\right) \tag{450}$$

$$q_{Cu} = 7.126\times10^4\ C \tag{451}$$

The time-forward velocity v_x is,

$$v_x = \frac{x}{t} \tag{452}$$

The time-forward current I_0 flowing through a copper wire is,

$$I_0 = \frac{q_{Cu}}{t} = \frac{n_{Cu}\,\mathcal{V}\,e^-}{x/v_x} = \pi\, r^2 n_{Cu}\, e^-\, v_x = \pi\, r^2 n_{Cu}\, e^-\, v_x = 10.0\ Amps \tag{453}$$

So, the time-forward drift velocity v_x of an electron moving through a copper wire is,

$$v_x = \frac{I_0}{\pi\, r^2 n_{Cu}\, e^-} = \frac{J_0}{n_{Cu}\, e^-} = \frac{\left(1.901\times10^6\ Amps/m^2\right)}{\left(8.4533\times10^{28}\ electrons/m^3\right)\left(1.60217733\times10^{-19}\,C\right)} \tag{454}$$

$$v_x = 1.403\times10^{-4}\ m/\sec \tag{455}$$

The POSITIVE fluctuating mass ΔM_e of the electron e^- is,

$$\Delta M_e = M_e\,\frac{{v_x}^2}{2c^2} = \left(9.1093897\times10^{-31}\,kg\right)\frac{\left(1.403\times10^{-4}\ m/\sec\right)^2}{2\left(2.99792458\times10^8\ m/\sec\right)^2} \tag{456}$$

$$\Delta M_e = 9.980\times10^{-56}\,kg \tag{457}$$

The POSITIVE fluctuating mass of an electron is almost 25 orders of magnitude below its' rest mass M_e.

Applying the new **Principle of Equivalence Theorem**,

$$y_1 = \frac{y_0}{1+\dfrac{y_0\,{v_x}^2}{2\,G\,M_E}} = \frac{\left(6.3781\times10^6\,m\right)}{1+\dfrac{\left(6.3781\times10^6\,m\right)\left(1.403\times10^{-4}\ m/\sec\right)^2}{2\left(6.67260\times10^{-11}\ N\,m^2/kg^2\right)\left(5.9787\times10^{24}\ kg\right)}} \tag{458}$$

$$y_1 = 6.3781\times10^6\,m \tag{459}$$

So, the equivalent POSITIVE displacement Δy is gravitational within the Earth's gravity well is,

$$\Delta y = y_0 - y_1 = 1.8627\times10^{-9}\,m \tag{460}$$

Given a time-forward voltage V and a time-forward current I, the resistivity ρ_{Cu} of copper wire is given as,

$$\rho_{Cu} = \frac{E}{J} = \frac{V/x}{I/A} = \frac{V/x}{I/\pi r^2} = 1.68\times10^{-8}\,\Omega\,m \qquad (461)$$

The resistance R of a segment of copper wire is,

$$R = \frac{V}{I} = \rho_{Cu}\frac{x}{A} = \left(1.68\times10^{-8}\,\Omega\,m\right)\frac{(1m)}{\pi\left(1.294\times10^{-3}\,m\right)^2} = 3.193\times10^{-3}\,\Omega \qquad (462)$$

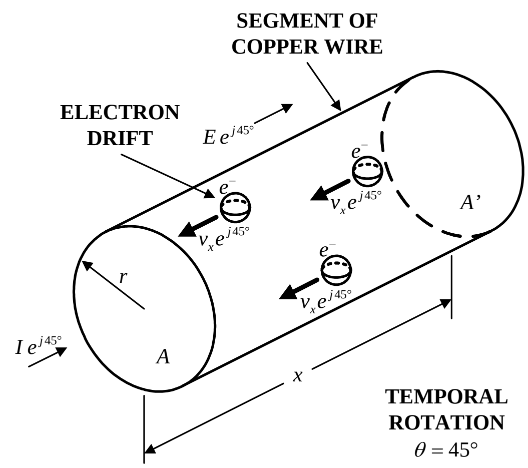


FIGURE 35. The time-advanced electron drift velocity in a copper wire.

Example 13. A time-advanced electric current I is established if a net charge q_{Cu} passes through any cross sectional area A of the conductor in time-advanced t. The electrons move at an average time-advanced drift velocity v_x.

So, given,

Direction of time $\theta = 45°$

Current flow through copper wire $I = 10.0\,Amps$

Fundamental charge of an electron $e^- = 1.602177\times10^{-19}\,C$

Radius of $10AWG$ copper wire $r = 1.294\times10^{-3}\,m$

Density of copper conductor $(20°C)$ $D_{Cu} = 8.92\times10^{6}\,gm/m^3$

Number of conduction electrons per atom of copper $k_{Cu} = 1\,electron/atom$

Avogadro's Number $N_0 = 6.0221367\times10^{23}\,atoms/mole$

Atomic weight of copper conductor $W_{Cu} = 63.546\,gm/mole$

Segment length $x = 1m$

Speed of light $c = 2.99792458\times10^{8}\,m/\sec$

Rest mass of an electron $M_e = 9.1093897\times10^{-31}\,kg$

Gravitational constant $G = 6.67260\times10^{-11}\,N\,m^2/kg^2$

Mean Radius of surface of Earth $y_0 = 6.3781\times10^{6}\,m$

Mass of the Earth $M_E = 5.9787\times10^{24}\,kg$

Resistivity of copper conductor $(20°C)$ $\rho_{Cu} = 1.68\times10^{-8}\,\Omega\,m$

The time-advanced current I_0 flowing through a conductor, where $\theta = 45°$ is,

$$I_0 = I\,e^{j\theta} = (10.0\,Amps)\,e^{j45°} = 7.071 + 7.071\,j\;Amps \tag{463}$$

The time-advanced current density J_0 is,

$$J_0 = \frac{I_0}{A} = \frac{I_0}{\pi r^2} = \frac{(7.071 + 7.071\,j\;Amps)}{\pi\left(1.294\times10^{-3}\,m\right)^2} = 1.344\times10^6 + 1.344\,j\times10^6\;Amps/m^2 \tag{464}$$

The volume $\mathcal{V}$ of a segment of copper wire is,

$$\mathcal{V} = A\,x = \pi r^2 x = \pi\left(1.294\times10^{-3}\,m\right)^2 (1m) = 5.261\times10^{-6}\,m^3 \tag{465}$$

The quantity of conduction electrons n_{Cu} in a volume of copper wire is,

$$n_{Cu} = \frac{D_{Cu}\,N_0\,k_{Cu}}{W_{Cu}} \tag{466}$$

$$n_{Cu} = \frac{\left(8.92\times10^6\;gm/m^3\right)\left(6.0221367\times10^{23}\;atoms/mole\right)\left(1\,electron/atom\right)}{\left(63.546\;gm/mole\right)} \tag{467}$$

$$n_{Cu} = 8.4533\times10^{28}\;electrons/m^3 \tag{468}$$

The net charge q_{Cu} in a volume of a copper wire is,

$$q_{Cu} = n_{Cu}\,\mathcal{V}\,e^- = \left(8.4533\times10^{28}\;electrons/m^3\right)\left(5.261\times10^{-6}\,m^3\right)\left(1.60217733\times10^{-19}\,C\right) \tag{469}$$

$$q_{Cu} = 7.126\times10^4\;C \tag{470}$$

The time-advanced velocity v_x is,

$$v_x = \frac{x}{t} \tag{471}$$

The time-advanced current I_0 flowing through a copper wire is,

$$I_0 = \frac{q_{Cu}}{t} = \frac{n_{Cu}\,\mathcal{V}\,e^-}{x/v_x} = \pi r^2 n_{Cu}\,e^-\,v_x = 7.071 + 7.071\,j\;Amps \tag{472}$$

So, the time-advanced drift velocity v_x of an electron moving through a copper wire is,

$$v_x = \frac{I_0}{\pi r^2 n_{Cu}\,e^-} = \frac{J_0}{n_{Cu}\,e^-} = \frac{\left(1.344\times10^6 + 1.344\,j\times10^6\;Amps/m^2\right)}{\left(8.4533\times10^{28}\;electrons/m^3\right)\left(1.60217733\times10^{-19}\,C\right)} \tag{473}$$

$$v_x = 9.923\times10^{-5} + 9.923\,j\times10^{-5}\ m/\sec \tag{474}$$

The fluctuating mass ΔM_e of the electron e^- is,

$$\Delta M_e = M_e \frac{v_x^{\ 2}}{2c^2} = \left(9.1093897\times10^{-31}kg\right)\frac{\left(9.923\times10^{-5} + 9.923\,j\times10^{-5}\ m/\sec\right)^2}{2\left(2.99792458\times10^{8}\ m/\sec\right)^2} \tag{475}$$

$$\Delta M_e = 9.980\,j\times10^{-56}kg \tag{476}$$

The fluctuating mass of an electron is almost 25 orders of magnitude beyond its' rest mass M_e and is imaginary.

Applying the new **Principle of Equivalence Theorem**,

$$y_1 = \frac{y_0}{1+\dfrac{y_0\,v_x^{\ 2}}{2\,G\,M_E}} = \frac{\left(6.3781\times10^{6}m\right)}{1+\dfrac{\left(6.3781\times10^{6}m\right)\left(9.923\times10^{-5}+9.923\,j\times10^{-5}\ m/\sec\right)^2}{2\left(6.67260\times10^{-11}\ N\,m^2/kg^2\right)\left(5.9787\times10^{24}\ kg\right)}} \tag{477}$$

$$y_1 = 6.3781\times10^{6}m \tag{478}$$

So, the equivalent IMAGINARY displacement Δy is shown to be non-gravitational within the Earth's gravity well is,

$$\Delta y = y_0 - y_1 = 1.0041\,j\times10^{-9}m \tag{479}$$

Given a time-advanced voltage V and a time-advanced current I, the resistivity ρ_{Cu} of copper wire is given as,

$$\rho_{Cu} = \frac{E}{J} = \frac{V/x}{I/A} = \frac{V/x}{I/\pi r^2} = 1.68\times10^{-8}\,\Omega\,m \tag{480}$$

The resistance R of a segment of copper wire is,

$$R = \frac{V}{I} = \rho_{Cu}\frac{x}{A} = \left(1.68\times10^{-8}\,\Omega\,m\right)\frac{\left(1m\right)}{\pi\left(1.294\times10^{-3}m\right)^2} = 3.193\times10^{-3}\,\Omega \tag{481}$$

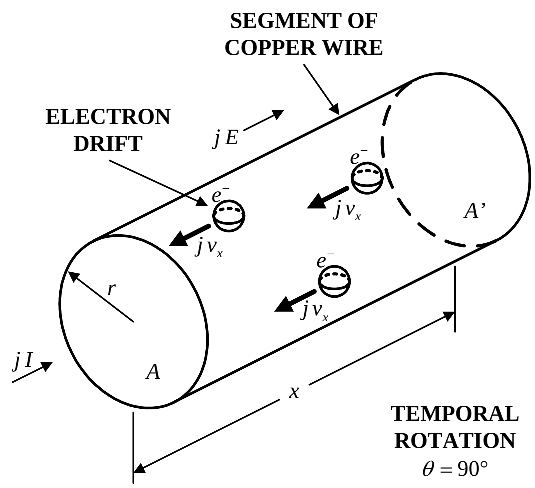


FIGURE 36. The time-future electron drift velocity in a copper wire.

Example 14. A time-future electric current $+jI$ is established if a net charge q_{Cu} passes through any cross sectional area A of the conductor in time-future t. The electrons move at an average time-future drift velocity $+jv_x$.

So, given,

Direction of time $\theta = 90°$

Current flow through copper wire $I = 10.0\,Amps$

Fundamental charge of an electron $e^- = 1.602177 \times 10^{-19}\,C$

Radius of $10AWG$ copper wire $r = 1.294 \times 10^{-3}\,m$

Density of copper conductor $(20°C)$ $D_{Cu} = 8.92 \times 10^6\ gm/m^3$

Number of conduction electrons per atom of copper $k_{Cu} = 1\ electron/atom$

Avogadro's Number $N_0 = 6.0221367 \times 10^{23}\ atoms/mole$

Atomic weight of copper conductor $W_{Cu} = 63.546\ gm/mole$

Segment length $x = 1m$

Speed of light $c = 2.99792458 \times 10^8\ m/\sec$

Rest mass of an electron $M_e = 9.1093897 \times 10^{-31}\,kg$

Gravitational constant $G = 6.67260 \times 10^{-11}\ N\,m^2/kg^2$

Mean Radius of surface of Earth $y_0 = 6.3781 \times 10^6\,m$

Mass of the Earth $M_E = 5.9787 \times 10^{24}\ kg$

Resistivity of copper conductor $(20°C)$ $\rho_{Cu} = 1.68 \times 10^{-8}\,\Omega\,m$

The time-future current I_0 flowing through a conductor, where $\theta = 90°$ is,

$$I_0 = I\,e^{j\theta} = (10.0\,Amps)\,e^{j90°} = 10.0\,j\,Amps \tag{482}$$

The time-future current density J_0 is,

$$J_0 = \frac{I_0}{A} = \frac{I_0}{\pi r^2} = \frac{(10.0\,j\,Amps)}{\pi\left(1.294 \times 10^{-3}\,m\right)^2} = 1.901\,j \times 10^6\ Amps/m^2 \tag{483}$$

The volume $\mathcal{V}$ of a segment of copper wire is,

$$\mathcal{V} = A\,x = \pi\,r^2 x = \pi\left(1.294\times10^{-3}\,m\right)^2\left(1m\right) = 5.261\times10^{-6}\,m^3 \tag{484}$$

The quantity of conduction electrons n_{Cu} in a volume of copper wire is,

$$n_{Cu} = \frac{D_{Cu}\;N_0\;k_{Cu}}{W_{Cu}} \tag{485}$$

$$n_{Cu} = \frac{\left(8.92\times10^6\;gm/m^3\right)\left(6.0221367\times10^{23}\;atoms/mole\right)\left(1electron/atom\right)}{\left(63.546\,gm/mole\right)} \tag{486}$$

$$n_{Cu} = 8.4533\times10^{28}\;electrons/m^3 \tag{487}$$

The net charge q_{Cu} in a volume of a copper wire is,

$$q_{Cu} = n_{Cu}\,\mathcal{V}\,e^- = \left(8.4533\times10^{28}\;electrons/m^3\right)\left(5.261\times10^{-6}\,m^3\right)\left(1.60217733\times10^{-19}\,C\right) \tag{488}$$

$$q_{Cu} = 7.126\times10^4\;C \tag{489}$$

The time-future velocity v_x is,

$$v_x = \frac{x}{t} \tag{490}$$

The time-future current I_0 flowing through a copper wire is,

$$I_0 = \frac{q_{Cu}}{t} = \frac{n_{Cu}\,\mathcal{V}\,e^-}{x/v_x} = \pi\,r^2 n_{Cu}\;e^-\;v_x = \pi\,r^2 n_{Cu}\;e^-\;v_x = 10.0\,j\;Amps \tag{491}$$

So, the time-future drift velocity v_x of an electron moving through a copper wire is,

$$v_x = \frac{I_0}{\pi\,r^2 n_{Cu}\,e^-} = \frac{J_0}{n_{Cu}\,e^-} = \frac{\left(1.901\,j\times10^6\;Amps/m^2\right)}{\left(8.4533\times10^{28}\;electrons/m^3\right)\left(1.60217733\times10^{-19}\,C\right)} \tag{492}$$

$$v_x = 1.403\,j\times10^{-4}\;m/\mathrm{sec} \tag{493}$$

The NEGATIVE fluctuating mass ΔM_e of the electron e^- is,

$$\Delta M_e = M_e\,\frac{{v_x}^2}{2\,c^2} = \left(9.1093897\times10^{-31}\,kg\right)\frac{\left(1.403\,j\times10^{-4}\;m/\mathrm{sec}\right)^2}{2\left(2.99792458\times10^8\;m/\mathrm{sec}\right)^2} \tag{494}$$

$$\Delta M_e = -9.980\times10^{-56}\,kg \tag{495}$$

The NEGATIVE fluctuating mass of an electron is almost 25 orders of magnitude below its' rest mass M_e.

Applying the new **Principle of Equivalence Theorem**,

$$y_1 = \frac{y_0}{1 + \frac{y_0\, v_x^{\,2}}{2\,G\,M_E}} = \frac{\left(6.3781\times 10^{6}\, m\right)}{1 + \frac{\left(6.3781\times 10^{6}\, m\right)\left(1.403\, j \times 10^{-4}\; m/\sec\right)^2}{2\left(6.67260\times 10^{-11}\; N\, m^2 / kg^2\right)\left(5.9787\times 10^{24}\; kg\right)}} \tag{496}$$

$$y_1 = 6.3781\times 10^{6}\, m \tag{497}$$

So, the equivalent NEGATIVE displacement Δy is antigravitational within the Earth's gravity well is,

$$\Delta y = y_0 - y_1 = -9.3132\times 10^{-10}\, m \tag{498}$$

Given a time-future voltage V and a time-future current I, the resistivity ρ_{Cu} of copper wire is given as,

$$\rho_{Cu} = \frac{E}{J} = \frac{V/x}{I/A} = \frac{V/x}{I/\pi\, r^2} = 1.68\times 10^{-8}\,\Omega\, m \tag{499}$$

The resistance R of a segment of copper wire is,

$$R = \frac{V}{I} = \rho_{Cu}\,\frac{x}{A} = \left(1.68\times 10^{-8}\,\Omega\, m\right)\frac{\left(1 m\right)}{\pi\left(1.294\times 10^{-3}\, m\right)^2} = 3.193\times 10^{-3}\,\Omega \tag{500}$$

COMPLEX RESISTOR

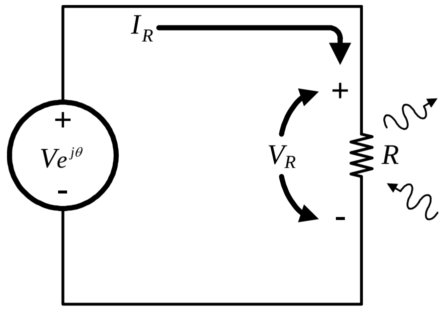


FIGURE 37. The complete *complex* resistor.

Given a *complex* voltage source V_S with a temporal rotation operator $e^{j\theta}$, where $0° \le \theta \le 90°$ is acting upon the voltage, a *complex* direct current flows through resistor R. A *complex* voltage V_R appears across the resistor. The resulting instantaneous power P_R is dissipated or absorbed by the resistor.

So, given,
- Direction of time θ
- Voltage supply V
- Resistor R

The *complex* voltage supply V_S is,

$$V_S = V\,e^{j\theta} = V\cos\theta + jV\sin\theta \tag{501}$$

The *complex* current I_R flowing through resistor R is,

$$I_R = \frac{V_S}{R} \tag{502}$$

The *complex* voltage V_R across resistor R is,

$$V_R = I_R\,R = V_S \tag{503}$$

The resistance R is,

$$R = \frac{V_S}{I_R} \tag{504}$$

The instantaneous power P_R dissipated and/or absorbed by the resistor is,

$$P_R = V_R\,I_R = I_R^{\,2}R = \frac{V_R^{\,2}}{R} \tag{505}$$

Example 15. Given a time-forward voltage source V_S and a known resistor value R, compute the time-forward current and power dissipated by the resistor.

So, given,
Direction of time $\theta = 0°$
Voltage Source $V = 10.0 Volts$
Resistor $R = 2.5\Omega$

The time-forward voltage V_S is,

$$V_S = V\,e^{j\theta} = (10.0 Volts)\,e^{j0°} = 10.0 Volts \tag{506}$$

The time-forward current I_R is,

$$I_R = \frac{V_S}{R} = \frac{(10.0 Volts)}{(2.5\Omega)} = 4.0\,Amps \tag{507}$$

The instantaneous power P_R dissipated by the resistor is,

$$P_R = \frac{V_R^{\;2}}{R} = \frac{(10.0 Volts)^2}{(2.5\Omega)} = 40.0 Watts \tag{508}$$

Example 16. Given a time-advanced voltage source V_S and a known resistor value R, compute the time-advanced current and power being dissipated and absorbed by the resistor.

So, given,
Direction of time $\theta = 45°$
Voltage Source $V = 10.0 Volts$
Resistor $R = 2.5\Omega$

The time-advanced voltage V_S is,

$$V_S = V\,e^{j\theta} = (10.0 Volts)\,e^{j45°} = 7.0711 + 7.0711\,j\,Volts \tag{509}$$

The time-advanced current I_R is,

$$I_R = \frac{V_S}{R} = \frac{(7.0711 + 7.0711\,j\,Volts)}{(2.5\Omega)} = 2.8284 + 2.8284\,j\,Amps \tag{510}$$

The instantaneous power P_R dissipated and absorbed of the resistor is,

$$P_R = \frac{V_R^{\;2}}{R} = \frac{(7.0711 + 7.0711\,j\,Volts)^2}{(2.5\Omega)} = 40.0\,j\,Watts \tag{511}$$

The resistor is dissipating and absorbing an equal amount of heat. The resistor is therefore, temperature neutral or adiabatic.

Example 17. Given a time-future voltage source V_S and a known resistor value R, compute the time-future current and instantaneous power absorbed by the resistor.

So, given,
Direction of time $\theta = 90°$
Voltage Source $V = 10.0 Volts$
Resistor $R = 2.5\Omega$

The time-future voltage V_S is,

$$V_S = V e^{j\theta} = (10.0 Volts) e^{j90°} = 10.0 j Volts \tag{512}$$

The time-future current I_R is,

$$I_R = \frac{V_S}{R} = \frac{(10.0 j Volts)}{(2.5\Omega)} = 4.0 j\ Amps \tag{513}$$

The instantaneous power P_R absorbed by the resistor is,

$$P_R = \frac{V_R^{\ 2}}{R} = \frac{(10.0\ j Volts)^2}{(2.5\Omega)} = -40.0 Watts \tag{514}$$

COMPLEX INDUCTOR

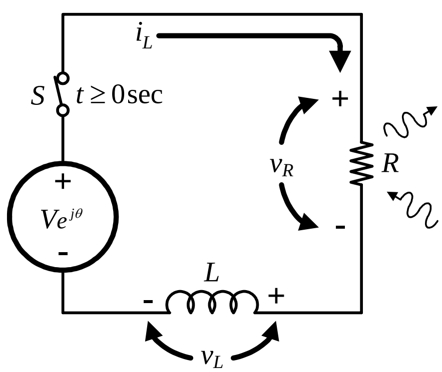


FIGURE 38. The complete *complex* magnetizing inductor.

Given a *complex* voltage source V_S with a temporal rotation operator $e^{j\theta}$, where $0° \leq \theta \leq 90°$ is acting upon the voltage, when switch S closes at $t = 0\sec$, a *complex* direct current i_L flows through resistor R and magnetizes inductor L. A *complex* voltage v_R appears across the resistor and a *complex* voltage v_L appears across inductor L. The resulting instantaneous power P_R is dissipated and/or absorbed by the resistor, the instantaneous power P_L stored in the inductor, and the energy E_L stored in the inductor.

So, given,
Direction of time θ
Time t
Voltage supply V
Inductor L

Resistor R

The *complex* voltage supply V_S is,

$$V_S = V\,e^{j\theta} = V\cos\theta + jV\sin\theta \tag{515}$$

The *complex* voltage across the resistor R is,

$$v_R(t) = i_L(t)R \tag{516}$$

The *complex* voltage across the inductor L is,

$$v_L(t) = L\frac{di_L}{dt} \tag{517}$$

Letting $t_0 = 0\sec$, the *complex* current i_L flowing through the resistor R and the inductor L is,

$$V_S = v_R(t) + v_L(t) = i_L(t)R + L\frac{di_L}{dt} \tag{518}$$

$$\frac{V_S}{R} = i_L(t) + \frac{L}{R}\frac{di_L}{dt} \tag{519}$$

$$i_L(t) = \frac{V_S}{R} - \frac{L}{R}\frac{di_L}{dt} \tag{520}$$

$$\int_0^{i_L(t)} \frac{1}{i_L(t) - \frac{V_S}{R}}\,di_L = -\frac{R}{L}\int_{t_0}^{t} dt \tag{521}$$

$$\ln\left(i_L(t) - \frac{V_S}{R}\right)\Bigg|_0^{i_L(t)} = -\frac{R}{L}t\Big|_{t_0}^{t} \tag{522}$$

$$\ln\left(i_L(t) - \frac{V_S}{R}\right) - \ln\left(-\frac{V_S}{R}\right) = \ln\left(\frac{i_L(t) - \frac{V_S}{R}}{-\frac{V_S}{R}}\right) = -\frac{R}{L}(t - t_0) \tag{523}$$

$$\frac{i_L(t) - \frac{V_S}{R}}{-\frac{V_S}{R}} = e^{-\frac{R}{L}(t-t_0)} = e^{-\frac{R}{L}t} \tag{524}$$

$$i_L(t) = \frac{V_S}{R} - \frac{V_S}{R}e^{-\frac{R}{L}t} = \frac{V_S}{R}\left(1 - e^{-\frac{R}{L}t}\right) \tag{525}$$

The instantaneous power P_R dissipated and/or absorbed by the resistor R is,

$$P_R(t) = v_R(t)\,i_L(t) = R\,i_L^{\,2}(t) = \frac{V_S^{\,2}}{R}\left(1 - e^{-\frac{R}{L}t}\right)^2 \tag{526}$$

The instantaneous power P_L stored in the inductor L is,

$$P_L(t) = v_L(t)\,i_L(t) = L\,i_L\frac{di_L}{dt} = V_S\,i_L(t) - R\,i_L^{\,2}(t) \tag{527}$$

$$P_L(t) = \frac{V_S^{\,2}}{R}\left(1 - e^{-\frac{R}{L}t}\right) - \frac{V_S^{\,2}}{R}\left(1 - e^{-\frac{R}{L}t}\right)^2 = \frac{V_S^{\,2}}{R}\left(1 - e^{-\frac{R}{L}t}\right)\left(1 - \left(1 - e^{-\frac{R}{L}t}\right)\right) = \frac{V_S^{\,2}}{R}\left(e^{-\frac{R}{L}t} - e^{-\frac{2R}{L}t}\right) \tag{528}$$

Letting $t_0 = 0\sec$, the energy E_L stored in the inductor L is,

$$E_L(t) = \int_{t_0}^{t} P_L\,dt = L\int_{t_0}^{t} i_L\frac{di_L}{dt}dt = L\int_{i(t_0)}^{i(t)} i_L\,di_L = \frac{1}{2}L\left[\left[i_L(t)\right]^2 - \left[i_L(t_0)\right]^2\right] \tag{529}$$

$$E_L(t) = \frac{1}{2}L\left[\left[\frac{V_S}{R}\left(1 - e^{-\frac{R}{L}t}\right)\right]^2 - \left[\frac{V_S}{R}\left(1 - e^{-\frac{R}{L}t_0}\right)\right]^2\right] = \frac{L V_S^{\,2}}{2R^2}\left[\left(1 - e^{-\frac{R}{L}t}\right)^2 - \left(1 - e^{-\frac{R}{L}t_0}\right)^2\right] \tag{530}$$

$$E_L(t) = \frac{L V_S^{\,2}}{2R^2}\left(1 - e^{-\frac{R}{L}t}\right)^2 \tag{531}$$

Example 18. Given a time-forward voltage source V_S, a known resistor value R and inductor value L, compute the time-forward current and power dissipated by the resistor, and the energy stored in the inductor.

So, given,

- Direction of time $\theta = 0°$
- Time $0.0\sec \le t \le 1.0\sec$
- Voltage supply $V = 10.0\,Volts$
- Inductor $L = 470\,mH$
- Resistor $R = 2.5\,\Omega$

The time-forward voltage supply V_S is,

$$V_S = V\,e^{j\theta} = (10.0\,Volts)\,e^{j0°} = 10.0\,Volts \tag{532}$$

The time-forward current i_L flowing through the resistor R and the inductor L at $t = 1.0\sec$ is,

$$i_L(t) = \frac{V_S}{R}\left(1 - e^{-\frac{R}{L}t}\right) = \frac{(10.0\,Volts)}{(2.5\,\Omega)}\left(1 - e^{-\frac{(2.5\Omega)}{(470\,mH)}t}\right) \tag{533}$$

$$i_L(1.0\sec) = 3.980\,Amps \tag{534}$$

The instantaneous power P_R dissipated by the resistor R at $t = 1.0\sec$ is,

$$P_R(t) = \frac{V_S{}^2}{R}\left(1 - e^{-\frac{R}{L}t}\right)^2 = \frac{(10.0\,Volts)^2}{(2.5\Omega)}\left(1 - e^{-\frac{(2.5\Omega)}{(470\,mH)}t}\right)^2 \tag{535}$$

$$P_R(1.0\sec) = 39.609\,Watts \tag{536}$$

The instantaneous power P_L stored in the inductor L at $t = 0.130\sec$ and at $t = 1.0\sec$ is,

$$P_L(t) = \frac{V_S{}^2}{R}\left(e^{-\frac{R}{L}t} - e^{-\frac{2R}{L}t}\right) = \frac{(10.0\,Volts)^2}{(2.5\Omega)}\left(e^{-\frac{(2.5\Omega)}{(470\,mH)}t} - e^{-\frac{2(2.5\Omega)}{(470\,mH)}t}\right) \tag{537}$$

$$P_L(0.130\sec) = 10.0\,Watts \tag{538}$$

$$P_L(1.0\sec) = 0.195\,Watts \tag{539}$$

The energy E_L stored in the inductor L from $t_0 = 0.0\sec$ to $t = 1.0\sec$ is,

$$E_L(t) == \frac{L\,V_S{}^2}{2\,R^2}\left(1 - e^{-\frac{R}{L}t}\right)^2 = \frac{(470\,mH)(10.0\,Volts)^2}{2(2.5\Omega)^2}\left(1 - e^{-\frac{(2.5\Omega)}{(470\,mH)}t}\right)^2 \tag{540}$$

$$E_L(1.0\sec) = 3.723\,Joules \tag{541}$$

Example 19. Given a time-advanced voltage source V_S, a known resistor value R and inductor value L, compute the time-advanced current and power dissipated and absorbed by the resistor, and the energy stored in the inductor.

So, given,
- Direction of time $\theta = 45°$
- Time $0.0\sec \le t \le 1.0\sec$
- Voltage supply $V = 10.0\,Volts$
- Inductor $L = 470\,mH$
- Resistor $R = 2.5\,\Omega$

The time-advanced voltage supply V_S is,

$$V_S = V\,e^{j\theta} = (10.0\,Volts)\,e^{j45°} = 7.071 + 7.071\,j\,Volts \tag{542}$$

The time-advanced current i_L flowing through the resistor R and the inductor L at $t = 1.0\sec$ is,

$$i_L(t) = \frac{V_S}{R}\left(1 - e^{-\frac{R}{L}t}\right) = \frac{(7.071 + 7.071\,j\,Volts)}{(2.5\Omega)}\left(1 - e^{-\frac{(2.5\Omega)}{(470\,mH)}t}\right) \tag{543}$$

$$i_L(1.0\sec) = 2.815 + 2.815\,j\,Amps \tag{544}$$

The instantaneous power P_R dissipated and absorbed by the resistor R at $t = 1.0\sec$ is,

$$P_R(t) = \frac{V_S{}^2}{R}\left(1 - e^{-\frac{R}{L}t}\right)^2 = \frac{\left(7.071 + 7.071\,j\,Volts\right)^2}{\left(2.5\Omega\right)}\left(1 - e^{-\frac{(2.5\Omega)}{(470\,mH)}t}\right)^2 \quad (545)$$

$$P_R\left(1.0\sec\right) = 39.609\,j\,Watts \quad (546)$$

The instantaneous power P_L stored in the inductor L at $t = 0.130\sec$ and at $t = 1.0\sec$ is,

$$P_L(t) = \frac{V_S{}^2}{R}\left(e^{-\frac{R}{L}t} - e^{-\frac{2R}{L}t}\right) = \frac{\left(7.071 + 7.071\,j\,Volts\right)^2}{\left(2.5\Omega\right)}\left(e^{-\frac{(2.5\Omega)}{(470\,mH)}t} - e^{-\frac{2(2.5\Omega)}{(470\,mH)}t}\right) \quad (547)$$

$$P_L\left(0.130\sec\right) = 10.0\,j\,Watts \quad (548)$$

$$P_L\left(1.0\sec\right) = 0.195\,j\,Watts \quad (549)$$

The energy E_L stored in the inductor L from $t_0 = 0.0\sec$ to $t = 1.0\sec$ is,

$$E_L(t) == \frac{L\,V_S{}^2}{2\,R^2}\left(1 - e^{-\frac{R}{L}t}\right)^2 = \frac{\left(470\,mH\right)\left(7.071 + 7.071\,j\,Volts\right)^2}{2\left(2.5\Omega\right)^2}\left(1 - e^{-\frac{(2.5\Omega)}{(470\,mH)}t}\right)^2 \quad (550)$$

$$E_L\left(1.0\sec\right) = 3.723\,j\,Joules \quad (551)$$

Example 20. Given a time-future voltage source V_S, a known resistor value R and inductor value L, compute the time-future current and power absorbed by the resistor, and the negative energy stored in the inductor.

So, given,

Direction of time $\theta = 90°$
Time $0.0\sec \le t \le 1.0\sec$
Voltage supply $V = 10.0\,Volts$
Inductor $L = 470\,mH$
Resistor $R = 2.5\,\Omega$

The time-future voltage supply V_S is,

$$V_S = V\,e^{j\theta} = \left(10.0\,Volts\right)e^{j90°} = 10.0\,j\,Volts \quad (552)$$

The time-future i_L flowing through the resistor R and the inductor L at $t = 1.0\sec$ is,

$$i_L(t) = \frac{V_S}{R}\left(1 - e^{-\frac{R}{L}t}\right) = \frac{\left(10.0\,j\,Volts\right)}{\left(2.5\Omega\right)}\left(1 - e^{-\frac{(2.5\Omega)}{(470\,mH)}t}\right) \quad (553)$$

$$i_L\left(1.0\sec\right) = 3.980\,j\,Amps \quad (554)$$

The instantaneous power P_R absorbed by the resistor R at $t = 1.0\sec$ is,

$$P_R(t) = \frac{V_S^{\,2}}{R}\left(1 - e^{-\frac{R}{L}t}\right)^2 = \frac{(10.0\,j\,Volts)^2}{(2.5\Omega)}\left(1 - e^{-\frac{(2.5\Omega)}{(470\,mH)}t}\right)^2 \tag{555}$$

$$P_R(1.0\sec) = -39.609\,Watts \tag{556}$$

The instantaneous power P_L stored in the inductor L at $t = 0.130\sec$ and at $t = 1.0\sec$ is,

$$P_L(t) = \frac{V_S^{\,2}}{R}\left(e^{-\frac{R}{L}t} - e^{-\frac{2R}{L}t}\right) = \frac{(10.0\,j\,Volts)^2}{(2.5\Omega)}\left(e^{-\frac{(2.5\Omega)}{(470\,mH)}t} - e^{-\frac{2(2.5\Omega)}{(470\,mH)}t}\right) \tag{557}$$

$$P_L(0.130\sec) = -10.0\,Watts \tag{558}$$

$$P_L(1.0\sec) = -0.195\,Watts \tag{559}$$

The energy E_L stored in the inductor L from $t_0 = 0.0\sec$ to $t = 1.0\sec$ is,

$$E_L(t) = \frac{L V_S^{\,2}}{2R^2}\left(1 - e^{-\frac{R}{L}t}\right)^2 = \frac{(470\,mH)(10.0\,j\,Volts)^2}{2(2.5\Omega)^2}\left(1 - e^{-\frac{(2.5\Omega)}{(470\,mH)}t}\right)^2 \tag{560}$$

$$E_L(1.0\sec) = -3.723\,Joules \tag{561}$$

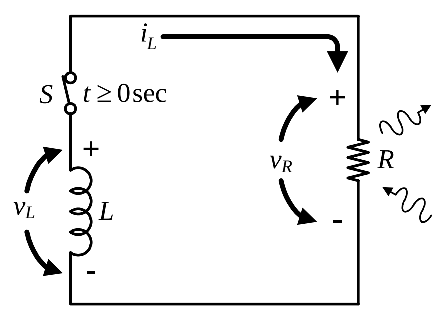


FIGURE 39. The complete *complex* demagnetizing inductor.

Given energy E_L stored in inductor L with a temporal rotation operator $e^{j\theta}$, where $0° \le \theta \le 90°$ is acting upon the voltage, when switch S closes at $t = 0\sec$, a *complex* direct current i_L flows through resistor R. The inductor L demagnetizes into the resistor. A *complex* voltage v_R appears across the resistor R. The resulting instantaneous power P_R and energy E_R are dissipated and/or absorbed by the resistor.

So, given,

- Direction of time θ
- Time t
- Initial current through inductor I
- Inductor L
- Resistor R

The *complex* current I_0 through the inductor L is,

$$I_0 = I\,e^{j\theta} = I\cos\theta + j\,I\sin\theta \tag{562}$$

At $t = 0\sec$, the voltage V_0 across the resistor R is,

$$V_0 = I_0\,R \tag{563}$$

The *complex* voltage v_R across the resistor R is,

$$v_R(t) = R\,i_L(t) \tag{564}$$

The *complex* voltage v_L across the inductor L is,

$$v_L(t) = -L\frac{di_L}{dt} \tag{565}$$

Letting $t_0 = 0\sec$, the *complex* current i_L flowing through the resistor R and the inductor L is,

$$v_R(t) = v_L(t) \tag{566}$$

$$i_L(t)\,R = -L\frac{di_L}{dt} \tag{567}$$

$$i_L(t) = -\frac{L}{R}\frac{di_L}{dt} \tag{568}$$

$$\frac{1}{i_L(t)}di_L = -\frac{R}{L}dt \tag{569}$$

$$\int_{I_0}^{i(t)}\frac{1}{i_L(t)}di_L = -\frac{R}{L}\int_{t_0}^{t}dt \tag{570}$$

$$\ln\left(i_L(t)\right)\Big|_{I_0}^{i_L(t)} = -\frac{R}{L}t\Big|_{t_0}^{t} \tag{571}$$

$$\ln\left(i_L(t)\right) - \ln\left(I_0\right) = \ln\left(\frac{i_L(t)}{I_0}\right) = -\frac{R}{L}\left(t - t_0\right) \tag{572}$$

$$\frac{i_L(t)}{I_0} = e^{-\frac{R}{L}\left(t-t_0\right)} \tag{573}$$

$$i_L(t) = I_0\,e^{-\frac{R}{L}t} \tag{574}$$

The instantaneous power P_R dissipated and/or absorbed by the resistor R is,

$$P_R(t) = v_R(t)\, i_L(t) = R\, i_L^{\,2}(t) = R\, I_0^{\,2} e^{-\frac{2R}{L}t} \tag{575}$$

Letting $t_0 = 0\sec$, the energy E_R dissipated and/or absorbed by the resistor R is,

$$E_R(t) = \int_{t_0}^{t} P_R\, dt = R\, I_0^{\,2} \int_{t_0}^{t} e^{-\frac{2R}{L}t} dt \tag{576}$$

$$E_R(t) = R\, I_0^{\,2}\left(\frac{-L}{2R}\right) e^{-\frac{2R}{L}t} - R\, I_0^{\,2}\left(\frac{-L}{2R}\right) e^{-\frac{2R}{L}t_0} == \frac{L\, I_0^{\,2}}{2}\left[e^{-\frac{2R}{L}t_0} - e^{-\frac{2R}{L}t}\right] \tag{577}$$

$$E_R(t) = \frac{L\, I_0^{\,2}}{2}\left[1 - e^{-\frac{2R}{L}t}\right] \tag{578}$$

Example 21. Given a time-forward voltage V across inductor L, a known resistor value R and inductor value L, compute the time-forward current flowing through the resistor, and the power and energy dissipated by the resistor.

So, given,

- Direction of time $\theta = 0°$
- Time $0.0\sec \le t \le 1.0\sec$
- Initial current through inductor $I = 4.0\, Amps$
- Inductor $L = 470\, mH$
- Resistor $R = 2.5\Omega$

The time-forward current I_0 through the inductor L is,

$$I_0 = I\, e^{j\theta} = (4.0\, Amps)\, e^{j0°} = 4.0\, Amps \tag{579}$$

The time-forward current i_L flowing through the resistor R and the inductor L at $t = 0.0\sec$ and at $t = 1.0\sec$ is,

$$i_L(t) = I_0\, e^{-\frac{R}{L}t} = (4.0\, Amps)\, e^{-\frac{(2.5\Omega)}{(470\, mH)}t} \tag{580}$$

$$i_L(0.0\sec) = 4.0\, Amps \tag{581}$$

$$i_L(1.0\sec) = 0.020\, Amps \tag{582}$$

The instantaneous power P_R dissipated by the resistor R at $t = 0.0\sec$ and at $t = 1.0\sec$ is,

$$P_R(t) = R\, I_0^{\,2}\, e^{-\frac{2R}{L}t} = (2.5\Omega)(4.0\, Amps)^2\, e^{-\frac{2(2.5\Omega)}{(470\, mH)}t} \tag{583}$$

$$P_R(0.0\sec) = 40.0\, Watts \tag{584}$$

$$P_R(1.0\sec) = 9.592 \times 10^{-4}\, Watts \tag{585}$$

The energy E_R dissipated by the resistor R at $t = 1.0\sec$ is,

$$E_R(t) = \frac{L I_0^2}{2}\left[1 - e^{-\frac{2R}{L}t}\right] = \frac{(470\,mH)(4.0\,Amps)^2}{2}\left(1 - e^{-\frac{2(2.5\Omega)}{(470\,mH)}t}\right) \tag{586}$$

$$E_R(1.0\sec) = 3.760\,Joules \tag{587}$$

Example 22. Given a time-advanced voltage V across inductor L, a known resistor value R and inductor value L, compute the time-advanced current flowing through the resistor, and the power and energy dissipated and absorbed by the resistor.

So, given,
- Direction of time $\theta = 45°$
- Time $0.0\sec \le t \le 1.0\sec$
- Initial current through inductor $I = 4.0\,Amps$
- Inductor $L = 470\,mH$
- Resistor $R = 2.5\Omega$

The time-advanced current I_0 through the inductor L is,

$$I_0 = I\,e^{j\theta} = (4.0\,Amps)e^{j45°} = 2.828 + 2.828\,j\,Amps \tag{588}$$

The time-advanced current i_L flowing through the resistor R and the inductor L at $t = 0.0\sec$ and at $t = 1.0\sec$ is,

$$i_L(t) = I_0\,e^{-\frac{R}{L}t} = (2.828 + 2.828\,j\,Amps)e^{-\frac{(2.5\Omega)}{(470\,mH)}t} \tag{589}$$

$$i_L(0.0\sec) = 2.828 + 2.828\,j\,Amps \tag{590}$$

$$i_L(1.0\sec) = 0.014 + 0.014\,j\,Amps \tag{591}$$

The instantaneous power P_R dissipated and absorbed by the resistor R at $t = 0.0\sec$ and at $t = 1.0\sec$ is,

$$P_R(t) = R\,I_0^2\,e^{-\frac{2R}{L}t} = (2.5\Omega)(2.828 + 2.828\,j\,Amps)^2\,e^{-\frac{2(2.5\Omega)}{(470\,mH)}t} \tag{592}$$

$$P_R(0.0\sec) = 40.0\,j\,Watts \tag{593}$$

$$P_R(1.0\sec) = 9.592\,j \times 10^{-4}\,Watts \tag{594}$$

The energy E_R dissipated and absorbed by the resistor R at $t = 0.0\sec$ and at $t = 1.0\sec$ is,

$$E_R(t) = \frac{L I_0^2}{2}\left[1 - e^{-\frac{2R}{L}t}\right] = \frac{(470\,mH)(2.828 + 2.828\,j\,Amps)^2}{2}\left(1 - e^{-\frac{2(2.5\Omega)}{(470\,mH)}t}\right) \tag{595}$$

$$E_R(1.0\sec) = 3.760\,j\,Joules \tag{596}$$

Example 23. Given a time-future voltage V across inductor L, a known resistor value R and inductor value L, compute the time-future current flowing through the resistor, and the power and energy absorbed by the resistor.

So, given,

- Direction of time $\theta = 90°$
- Time $0.0\sec \le t \le 1.0\sec$
- Initial current through inductor $I = 4.0\,Amps$
- Inductor $L = 470\,mH$
- Resistor $R = 2.5\Omega$

The time-future current I_0 through the inductor L is,

$$I_0 = I\,e^{j\theta} = (4.0\,Amps)\,e^{j90°} = 4.0\,j\,Amps \tag{597}$$

The time-future current i_L flowing through the resistor R and the inductor L at $t = 0.0\sec$ and at $t = 1.0\sec$ is,

$$i_L(t) = I_0\,e^{-\frac{R}{L}t} = (4.0\,j\,Amps)\,e^{-\frac{(2.5\Omega)}{(470\,mH)}t} \tag{598}$$

$$i_L(0.0\sec) = 4.0\,j\,Amps \tag{599}$$

$$i_L(1.0\sec) = 0.020\,j\,Amps \tag{600}$$

The instantaneous power P_R absorbed by the resistor R at $t = 0.0\sec$ and at $t = 1.0\sec$ is,

$$P_R(t) = R\,I_0^{\,2}\,e^{-\frac{2R}{L}t} = (2.5\Omega)(4.0\,j\,Amps)^2\,e^{-\frac{2(2.5\Omega)}{(470\,mH)}t} \tag{601}$$

$$P_R(0.0\sec) = -40.0\,Watts \tag{602}$$

$$P_R(1.0\sec) = -9.592\times10^{-4}\,Watts \tag{603}$$

The energy E_R absorbed by the resistor R at $t = 0.0\sec$ and at $t = 1.0\sec$ is,

$$E_R(t) = \frac{L\,I_0^{\,2}}{2}\left[1 - e^{-\frac{2R}{L}t}\right] = \frac{(470\,mH)(4.0\,j\,Amps)^2}{2}\left(1 - e^{-\frac{2(2.5\Omega)}{(470\,mH)}t}\right) \tag{604}$$

$$E_R(1.0\sec) = -3.760\,Joules \tag{605}$$

COMPLEX CAPACITOR

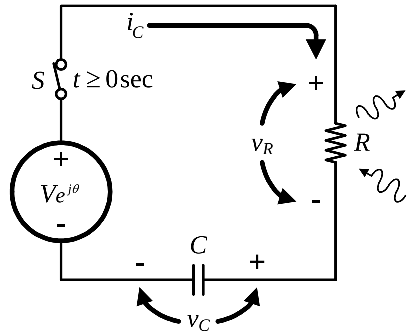


FIGURE 40. The complete *complex* charging capacitor.

Given a *complex* voltage source V_S with a temporal rotation operator $e^{j\theta}$, where $0° \leq \theta \leq 90°$ is acting upon the voltage, when switch S closes at $t = 0 \sec$, a *complex* direct current flows through resistor R and charges capacitor C. A *complex* voltage v_R appears across the resistor and a complex voltage v_C appears across capacitor C. The resulting power P_R is dissipated and/or absorbed by the resistor and the energy E_C is stored in the capacitor.

So, given,

- Direction of time θ
- Time t
- Voltage supply V
- Capacitor C
- Resistor R

The *complex* voltage supply V_S is,

$$V_S = V\, e^{j\theta} = V \cos\theta + j V \sin\theta \tag{606}$$

The *complex* voltage v_R across the resistor R is,

$$v_R(t) = i_C(t) R \tag{607}$$

The *complex* current i_C through a capacitor C is,

$$i_C(t) = C \frac{dv_C}{dt} \tag{608}$$

Letting $t_0 = 0 \sec$, the *complex* voltage v_C across the capacitor C is,

$$V_S = v_R(t) + v_C(t) = i_C(t) R + v_C(t) \tag{609}$$

$$v_C(t) = V_S - R C \frac{dv_C}{dt} \tag{610}$$

$$\frac{1}{v_C(t) - V_S} dv_C = -\frac{1}{RC} dt \tag{611}$$

$$\int_0^{v_C(t)} \frac{1}{v_C(t)-V_S} dv_C = -\frac{1}{RC}\int_{t_0}^{t} dt \tag{612}$$

$$\ln\left(v_C(t)-V_S\right)\Big|_0^{v_C(t)} = -\frac{1}{RC} t\Big|_{t_0}^{t} \tag{613}$$

$$\ln\left(v_C(t)-V_S\right)-\ln\left(-V_S\right)=\ln\left(\frac{v_C(t)-V_S}{-V_S}\right)=-\frac{1}{RC}\left(t-t_0\right) \tag{614}$$

$$\frac{v_C(t)-V_S}{-V_S}=e^{-\frac{1}{RC}(t-t_0)}=e^{-\frac{1}{RC}t} \tag{615}$$

$$v_C(t)=V_S-V_S\,e^{-\frac{1}{RC}t}=V_S\left(1-e^{-\frac{1}{RC}t}\right) \tag{616}$$

The instantaneous power P_R dissipated and/or absorbed by the resistor R is,

$$P_R(t)=v_R(t)i_C(t)=\frac{v_R^{\,2}(t)}{R}=\frac{\left(V_S-v_C(t)\right)^2}{R}=\frac{V_S^{\,2}}{R}e^{-\frac{2}{RC}t} \tag{617}$$

The instantaneous power P_C stored in the capacitor C is,

$$P_C(t)=v_C(t)i_C(t)=C\,v_C\frac{dv_C}{dt}=\frac{\left(V_S\,v_C(t)-v_C^{\,2}(t)\right)}{R} \tag{618}$$

$$P_C(t)=\frac{V_S^{\,2}}{R}\left(1-e^{-\frac{1}{RC}t}\right)-\frac{V_S^{\,2}}{R}\left(1-e^{-\frac{1}{RC}t}\right)^2=\frac{V_S^{\,2}}{R}\left(1-e^{-\frac{1}{RC}t}\right)\left(1-\left(1-e^{-\frac{1}{RC}t}\right)\right)=\frac{V_S^{\,2}}{R}\left(e^{-\frac{1}{RC}t}-e^{-\frac{2}{RC}t}\right) \tag{619}$$

Letting $t_0=0\sec$, the energy E_C stored in the capacitor C is,

$$E_L(t)=\int_{t_0}^{t} P_L\,dt=C\int_{t_0}^{t} v_C\frac{dv_C}{dt}dt=C\int_{v(t_0)}^{v(t)} v_C\,dv_C=\frac{1}{2}C\left[\left[v_C(t)\right]^2-\left[v_C(t_0)\right]^2\right] \tag{620}$$

$$E_C(t)=\frac{1}{2}C\left[\left[V_S\left(1-e^{-\frac{1}{RC}t}\right)\right]^2-\left[V_S\left(1-e^{-\frac{1}{RC}t_0}\right)\right]^2\right]=\frac{CV_S^{\,2}}{2}\left[\left(1-e^{-\frac{1}{RC}t}\right)^2-\left(1-e^{-\frac{1}{RC}t_0}\right)^2\right] \tag{621}$$

$$E_C(t)=\frac{CV_S^{\,2}}{2}\left(1-e^{-\frac{1}{RC}t}\right)^2 \tag{622}$$

Example 24. Given a time-forward voltage source V_S, a known resistor value R and capacitor value C, compute the time-forward current and power dissipated by the resistor, and the energy stored in the inductor.

So, given,
Direction of time $\theta=0°$

Time $0.0\sec \le t \le 1.0\sec$
Voltage supply $V = 10.0\,Volts$
Capacitor $C = 470\,\mu F$
Resistor $R = 1.0\,k\Omega$

The time-forward voltage supply V_S is,

$$V_S = V\,e^{j\theta} = \left(10.0\,Volts\right)e^{j0^\circ} = 10.0\,Volts \tag{623}$$

The time-forward voltage v_C across the capacitor C at $t = 1.0\sec$ is,

$$v_C\left(t\right) = V_S\left(1 - e^{-\frac{1}{RC}t}\right) = \left(10.0\,Volts\right)\left(1 - e^{-\frac{1}{(1.0\,k\Omega)(470\,\mu F)}t}\right) \tag{624}$$

$$v_C\left(1.0\sec\right) = 8.809\,Volts \tag{625}$$

The instantaneous power P_R dissipated by the resistor R at $t = 0.0\sec$ and at $t = 1.0\sec$ is,

$$P_R\left(t\right) = \frac{{V_S}^2}{R}e^{-\frac{2}{RC}t} = \frac{\left(10.0\,Volts\right)^2}{\left(1.0\,k\Omega\right)}e^{-\frac{2}{(1.0\,k\Omega)(470\,\mu F)}t} \tag{626}$$

$$P_R\left(0.0\sec\right) = 0.10\,Watts \tag{627}$$

$$P_R\left(1.0\sec\right) = 1.419\times 10^{-3}\,Watts \tag{628}$$

The instantaneous power P_C stored in the capacitor C at $t = 0.33\sec$ and at $t = 1.0\sec$ is,

$$P_C\left(t\right) = \frac{{V_S}^2}{R}\left(e^{-\frac{1}{RC}t} - e^{-\frac{2}{RC}t}\right) = \frac{\left(10.0\,Volts\right)^2}{\left(1.0\,k\Omega\right)}\left(e^{-\frac{1}{(1.0\,k\Omega)(470\,\mu F)}t} - e^{-\frac{2}{(1.0\,k\Omega)(470\,\mu F)}t}\right) \tag{629}$$

$$P_C\left(0.33\sec\right) = 0.0245\,Watts \tag{630}$$

$$P_C\left(1.0\sec\right) = 0.0105\,Watts \tag{631}$$

The energy E_C stored in the capacitor C from $t_0 = 0.0\sec$ to $t = 1.0\sec$ is,

$$E_C\left(t\right) = \frac{C\,{V_S}^2}{2}\left(1 - e^{-\frac{1}{RC}t}\right)^2 = \frac{\left(470\,\mu F\right)\left(10.0\,Volts\right)^2}{2}\left(1 - e^{-\frac{1}{(1.0\,k\Omega)(470\,\mu F)}t}\right)^2 \tag{632}$$

$$E_C\left(t\right) = 0.0182\,Joules \tag{633}$$

Example 25. Given a time-advanced voltage source V_S, a known resistor value R and capacitor value C, compute the time-advanced current and power dissipated and absorbed by the resistor, and the energy stored in the inductor.

So, given,

Direction of time $\theta = 45°$
Time $0.0\sec \le t \le 1.0\sec$
Voltage supply $V = 10.0\,Volts$
Capacitor $C = 470\,\mu F$
Resistor $R = 1.0\,k\Omega$

The time-advanced voltage supply V_S is,

$$V_S = V\,e^{j\theta} = (10.0\,Volts)\,e^{j45°} = 7.071 + 7.071\,j\,Volts \tag{634}$$

The time-advanced voltage v_C across the capacitor C at $t = 1.0\sec$ is,

$$v_C(t) = V_S\left(1 - e^{-\frac{1}{RC}t}\right) = (7.071 + 7.071\,j\,Volts)\left(1 - e^{-\frac{1}{(1.0\,k\Omega)(470\,\mu F)}t}\right) \tag{635}$$

$$v_C(1.0\sec) = 6.229 + 6.229\,j\,Volts \tag{636}$$

The instantaneous power P_R dissipated and absorbed by the resistor R at $t = 0.0\sec$ and at $t = 1.0\sec$ is,

$$P_R(t) = \frac{V_S^{\,2}}{R}e^{-\frac{2}{RC}t} = \frac{(7.071 + 7.071\,j\,Volts)^2}{(1.0\,k\Omega)}e^{-\frac{2}{(1.0\,k\Omega)(470\,\mu F)}t} \tag{637}$$

$$P_R(0.0\sec) = 0.10\,j\,Watts \tag{638}$$

$$P_R(1.0\sec) = 1.419\,j \times 10^{-3}\,Watts \tag{639}$$

The instantaneous power P_C stored in the capacitor C at $t = 0.33\sec$ and at $t = 1.0\sec$ is,

$$P_C(t) = \frac{V_S^{\,2}}{R}\left(e^{-\frac{1}{RC}t} - e^{-\frac{2}{RC}t}\right) = \frac{(7.071 + 7.071\,j\,Volts)^2}{(1.0\,k\Omega)}\left(e^{-\frac{1}{(1.0\,k\Omega)(470\,\mu F)}t} - e^{-\frac{2}{(1.0\,k\Omega)(470\,\mu F)}t}\right) \tag{640}$$

$$P_C(0.33\sec) = 0.0245\,j\,Watts \tag{641}$$

$$P_C(1.0\sec) = 0.0105\,Watts \tag{642}$$

The energy E_C stored in the capacitor C from $t_0 = 0.0\sec$ to $t = 1.0\sec$ is,

$$E_C(t) = \frac{C\,V_S^{\,2}}{2}\left(1 - e^{-\frac{1}{RC}t}\right)^2 = \frac{(470\,\mu F)(7.071 + 7.071\,j\,Volts)^2}{2}\left(1 - e^{-\frac{1}{(1.0\,k\Omega)(470\,\mu F)}t}\right)^2 \tag{643}$$

$$E_C(t) = 0.0182\,j\,Joules \tag{644}$$

Example 26. Given a time-future voltage source V_S, a known resistor value R and capacitor value C, compute the time-future current and power absorbed by the resistor, and the energy stored in the inductor.

So, given,

Direction of time $\theta = 90°$

Time $0.0\sec \le t \le 1.0\sec$

Voltage supply $V = 10.0\,Volts$

Capacitor $C = 470\,\mu F$

Resistor $R = 1.0\,k\Omega$

The time-future voltage supply V_S is,

$$V_S = V\,e^{j\theta} = \left(10.0\,Volts\right)e^{j90°} = 10.0\,j\,Volts \tag{645}$$

The time-future voltage v_C across the capacitor C at $t = 1.0\sec$ is,

$$v_C\left(t\right) = V_S\left(1 - e^{-\frac{1}{RC}t}\right) = \left(10.0\,j\,Volts\right)\left(1 - e^{-\frac{1}{(1.0\,k\Omega)(470\,\mu F)}t}\right) \tag{646}$$

$$v_C\left(1.0\sec\right) = 8.809\,j\,Volts \tag{647}$$

The instantaneous power P_R absorbed by the resistor R at $t = 0.0\sec$ and at $t = 1.0\sec$ is,

$$P_R\left(t\right) = \frac{V_S^{\,2}}{R}e^{-\frac{2}{RC}t} = \frac{\left(10.0\,j\,Volts\right)^2}{\left(1.0\,k\Omega\right)}e^{-\frac{2}{(1.0\,k\Omega)(470\,\mu F)}t} \tag{648}$$

$$P_R\left(0.0\sec\right) = -0.10\,Watts \tag{649}$$

$$P_R\left(1.0\sec\right) = -1.419\times 10^{-3}\,Watts \tag{650}$$

The instantaneous power P_C stored in the capacitor C at $t = 0.33\sec$ and at $t = 1.0\sec$ is,

$$P_C\left(t\right) = \frac{V_S^{\,2}}{R}\left(e^{-\frac{1}{RC}t} - e^{-\frac{2}{RC}t}\right) = \frac{\left(10.0\,j\,Volts\right)^2}{\left(1.0\,k\Omega\right)}\left(e^{-\frac{1}{(1.0\,k\Omega)(470\,\mu F)}t} - e^{-\frac{2}{(1.0\,k\Omega)(470\,\mu F)}t}\right) \tag{651}$$

$$P_C\left(0.33\sec\right) = -0.0245\,Watts \tag{652}$$

$$P_C\left(1.0\sec\right) = -0.0105\,Watts \tag{653}$$

The energy E_C stored in the capacitor C from $t_0 = 0.0\sec$ to $t = 1.0\sec$ is,

$$E_C\left(t\right) = \frac{C\,V_S^{\,2}}{2}\left(1 - e^{-\frac{1}{RC}t}\right)^2 = \frac{\left(470\,\mu F\right)\left(10.0\,j\,Volts\right)^2}{2}\left(1 - e^{-\frac{1}{(1.0\,k\Omega)(470\,\mu F)}t}\right)^2 \tag{654}$$

$$E_C\left(t\right) = -0.0182\,Joules \tag{655}$$

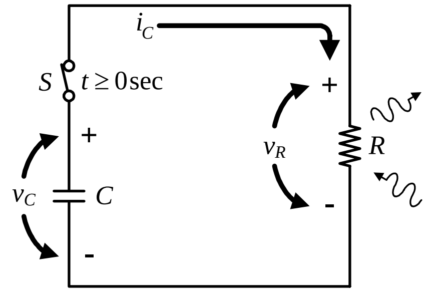


FIGURE 41. The complete *complex* discharging capacitor.

Given energy E_C stored in capacitor C with a temporal rotation operator $e^{j\theta}$, where $0° \leq \theta \leq 90°$ is acting upon the voltage, when switch S closes at $t = 0\sec$, a *complex* direct current i_C flows through resistor R. The capacitor C discharges into the resistor. A *complex* voltage v_R appears across the resistor R. The resulting instantaneous power P_R and energy E_R are dissipated and/or absorbed by the resistor.

So, given,
- Direction of time θ
- Time t
- Initial voltage across capacitor V
- Capacitor C
- Resistor R

At $t = 0\sec$, the voltage V_0 across the resistor R is,

$$V_0 = I_0 R \tag{656}$$

The *complex* voltage v_R across the resistor R is,

$$v_R(t) = R\, i_C(t) \tag{657}$$

The *complex* current i_C through a capacitor C is,

$$i_C(t) = -C\frac{dv_C}{dt} \tag{658}$$

Letting $t_0 = 0\sec$, the *complex* voltage v_C across the capacitor C is,

$$v_C(t) = v_R(t) = R\, i_C(t) = -RC\frac{dv_C}{dt} \tag{659}$$

$$\frac{1}{v_C(t)}dv_C = -\frac{1}{RC}dt \tag{660}$$

$$\int_{V_0}^{v_C(t)} \frac{1}{v_C(t)}dv_C = -\frac{1}{RC}\int_{t_0}^{t} dt \tag{661}$$

$$\ln\left(v_C(t)\right)\Big|_{V_0}^{v_C(t)} = -\frac{1}{RC}t\Big|_{t_0}^{t} \tag{662}$$

$$\ln\left(v_C(t)\right) - \ln\left(V_0\right) = \ln\left(\frac{v_C(t)}{V_0}\right) = -\frac{1}{RC}\left(t - t_0\right) \tag{663}$$

$$\frac{v_C(t)}{V_0} = e^{-\frac{1}{RC}\left(t-t_0\right)} = e^{-\frac{1}{RC}t} \tag{664}$$

$$v_C(t) = V_0\, e^{-\frac{1}{RC}t} \tag{665}$$

The instantaneous power P_R dissipated and/or absorbed by the resistor R is,

$$P_R(t) = v_R(t)\, i_C(t) = \frac{{v_R}^2(t)}{R} = \frac{{V_0}^2}{R} e^{-\frac{2}{RC}t} \tag{666}$$

Letting $t_0 = 0\sec$, the energy E_R dissipated and/or absorbed by the resistor R is,

$$E_R(t) = \int_{t_0}^{t} P_R\, dt = \frac{{V_0}^2}{R}\int_{t_0}^{t} e^{-\frac{2}{RC}t}\, dt \tag{667}$$

$$E_R(t) = \frac{{V_0}^2}{R}\left(\frac{-RC}{2}\right)e^{-\frac{2}{RC}t} - \frac{{V_0}^2}{R}\left(\frac{-RC}{2}\right)e^{-\frac{2}{RC}t_0} = \frac{1}{2}C{V_0}^2\left(e^{-\frac{2}{RC}t_0} - e^{-\frac{2}{RC}t}\right) \tag{668}$$

$$E_R(t) = \frac{1}{2}C{V_0}^2\left(1 - e^{-\frac{2}{RC}t}\right) \tag{669}$$

Example 27. Given a time-forward voltage V across capacitor C, a known resistor value R and capacitor value C, compute the time-forward current flowing through the resistor, and the power and energy dissipated by the resistor.

So, given,

- Direction of time $\theta = 0°$
- Time $0.0\sec \le t \le 1.0\sec$
- Initial voltage across capacitor $V = 10.0\,Volts$
- Capacitor $C = 470\,\mu F$
- Resistor $R = 1.0\,k\Omega$

The time-forward voltage V_0 across the capacitor C is,

$$V_0 = V\, e^{j\theta} = \left(10.0\,Volts\right)e^{j0°} = 10.0\,Volts \tag{670}$$

The time-forward voltage v_C across the capacitor C is,

$$v_C(t) = V_0\, e^{-\frac{1}{RC}t} = \left(10.0\,Volts\right)e^{-\frac{1}{(1.0\,k\Omega)(470\,\mu F)}t} \tag{671}$$

$$v_C\left(0.0\sec\right)=10.0\,Volts \tag{672}$$

$$v_C\left(1.0\sec\right)=1.191\,Volts \tag{673}$$

The instantaneous power P_R dissipated by the resistor R at $t=0.0\sec$ and at $t=1.0\sec$ is,

$$P_R\left(t\right)=\frac{V_0^{\ 2}}{R}e^{-\frac{2}{RC}t}=\frac{\left(10.0\,Volts\right)^2}{\left(1.0\,k\Omega\right)}e^{-\frac{2}{\left(1.0\,k\Omega\right)\left(470\,\mu F\right)}t} \tag{674}$$

$$P_R\left(0.0\sec\right)=0.10\,Watts \tag{675}$$

$$P_R\left(1.0\sec\right)=1.419\times10^{-3}\,Watts \tag{676}$$

The energy E_R dissipated by the resistor R at $t=0.0\sec$ and at $t=1.0\sec$ is,

$$E_R\left(t\right)=\frac{C\,V_0^{\ 2}}{2}\left(1-e^{-\frac{2}{RC}t}\right)=\frac{\left(470\,\mu F\right)\left(10.0\,Volts\right)^2}{2}\left(1-e^{-\frac{2}{\left(1.0\,k\Omega\right)\left(470\,\mu F\right)}t}\right) \tag{677}$$

$$E_R\left(1.0\sec\right)=0.023\,Joules \tag{678}$$

Example 28. Given a time-advanced voltage V across capacitor C, a known resistor value R and capacitor value C, compute the time-advanced current flowing through the resistor, and the power and energy dissipated and absorbed by the resistor.

So, given,

Direction of time $\theta=45°$

Time $0.0\sec\le t\le1.0\sec$

Initial voltage across capacitor $V=10.0\,Volts$

Capacitor $C=470\,\mu F$

Resistor $R=1.0\,k\Omega$

The time-advanced voltage V_0 across the capacitor C is,

$$V_0=V\,e^{j\theta}=\left(10.0\,Volts\right)e^{j45°}=7.071+7.071\,j\,Volts \tag{679}$$

The time-advanced voltage v_C across the capacitor C is,

$$v_C\left(t\right)=V_0\,e^{-\frac{1}{RC}t}=\left(7.071+7.071\,j\,Volts\right)e^{-\frac{1}{\left(1.0\,k\Omega\right)\left(470\,\mu F\right)}t} \tag{680}$$

$$v_C\left(0.0\sec\right)=7.071+7.071\,j\,Volts \tag{681}$$

$$v_C\left(1.0\sec\right)=0.842+0.842\,j\,Volts \tag{682}$$

The instantaneous power P_R dissipated and absorbed by the resistor R at $t = 0.0\sec$ and at $t = 1.0\sec$ is,

$$P_R(t) = \frac{V_0^{\,2}}{R} e^{-\frac{2}{RC}t} = \frac{\left(7.071 + 7.071\,j\,Volts\right)^2}{\left(1.0\,k\Omega\right)} e^{-\frac{2}{(1.0\,k\Omega)(470\,\mu F)}t} \tag{683}$$

$$P_R(0.0\sec) = 0.10\,j\,Watts \tag{684}$$

$$P_R(1.0\sec) = 1.419\,j \times 10^{-3}\,Watts \tag{685}$$

The energy E_R dissipated and absorbed by the resistor R at $t = 0.0\sec$ and at $t = 1.0\sec$ is,

$$E_R(t) = \frac{C V_0^{\,2}}{2}\left(1 - e^{-\frac{2}{RC}t}\right) = \frac{\left(470\,\mu F\right)\left(7.071 + 7.071\,j\,Volts\right)^2}{2}\left(1 - e^{-\frac{2}{(1.0\,k\Omega)(470\,\mu F)}t}\right) \tag{686}$$

$$E_R(1.0\sec) = 0.023\,j\;Joules \tag{687}$$

Example 29. Given a time-future voltage V across capacitor C, a known resistor value R and capacitor value C, compute the time-future current flowing through the resistor, and the power and energy absorbed by the resistor.

So, given,

- Direction of time $\theta = 90°$
- Time $0.0\sec \le t \le 1.0\sec$
- Initial voltage across capacitor $V = 10.0\,Volts$
- Capacitor $C = 470\,\mu F$
- Resistor $R = 1.0\,k\Omega$

The time-future voltage V_0 across the capacitor C is,

$$V_0 = V\,e^{j\theta} = \left(10.0\,Volts\right)e^{j90°} = 10.0\,j\,Volts \tag{688}$$

The time-future voltage v_C across the capacitor C is,

$$v_C(t) = V_0\,e^{-\frac{1}{RC}t} = \left(10.0\,j\,Volts\right)e^{-\frac{1}{(1.0\,k\Omega)(470\,\mu F)}t} \tag{689}$$

$$v_C(0.0\sec) = 10.0\,j\,Volts \tag{690}$$

$$v_C(1.0\sec) = 1.191\,j\,Volts \tag{691}$$

The instantaneous power P_R absorbed by the resistor R at $t = 0.0\sec$ and at $t = 1.0\sec$ is,

$$P_R(t) = \frac{V_0^{\,2}}{R} e^{-\frac{2}{RC}t} = \frac{\left(10.0\,j\,Volts\right)^2}{\left(1.0\,k\Omega\right)} e^{-\frac{2}{(1.0\,k\Omega)(470\,\mu F)}t} \tag{692}$$

$$P_R(0.0\sec) = -0.10\,Watts \tag{693}$$

$$P_R(1.0\sec) = -1.419 \times 10^{-3}\, Watts \tag{694}$$

The energy E_R absorbed by the resistor R at $t = 0.0\sec$ and at $t = 1.0\sec$ is,

$$E_R(t) = \frac{CV_0^{\,2}}{2}\left(1 - e^{-\frac{2}{RC}t}\right) = \frac{(470\,\mu F)(10.0\,j\,Volts)^2}{2}\left(1 - e^{-\frac{2}{(1.0\,k\Omega)(470\,\mu F)}t}\right) \tag{695}$$

$$E_R(1.0\sec) = -0.023\, Joules \tag{696}$$

COMPLEX FIELD MASS FLUCTUATION TECHNOLOGIES

Since a theoretical link was established between gravity and electromagnetism, two mass fluctuation technologies are presently under investigation. Both technologies are electrical devices with the first being inductive-based, and the second being capacitive-based. Shown below is a simplified schematic diagram that highlights their operation.

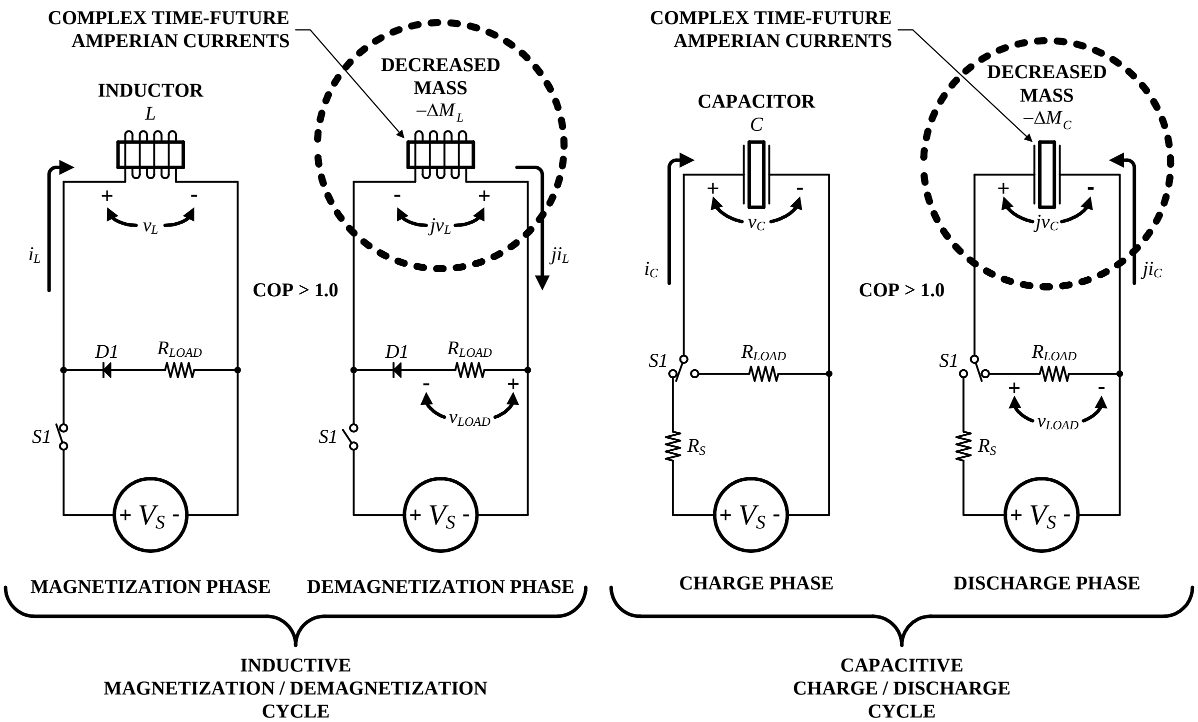


FIGURE 43. Two types of complex field mass fluctuating systems.

These systems are cyclic, and alter the local gravity well. The mass of these systems is converted to excess field energy during the magnetizing/charging phase. During the demagnetizing/discharging phase, excess electrical energy is collected, and mass is restored after this phase. Then, the cycle begins again. As a consequence, clocks runs faster due to broken symmetry of mass-energy conservation in the proximity of these devices because mass is converted to NEGATIVE energy.

AN INDICATOR OF EXCESS FREE ENERGY

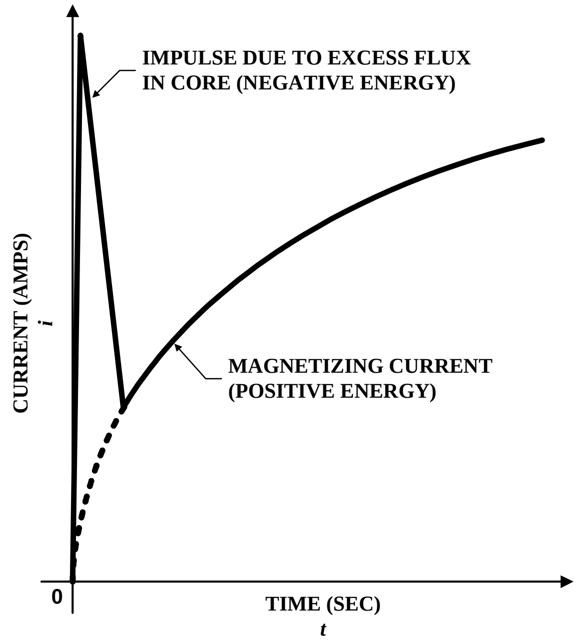
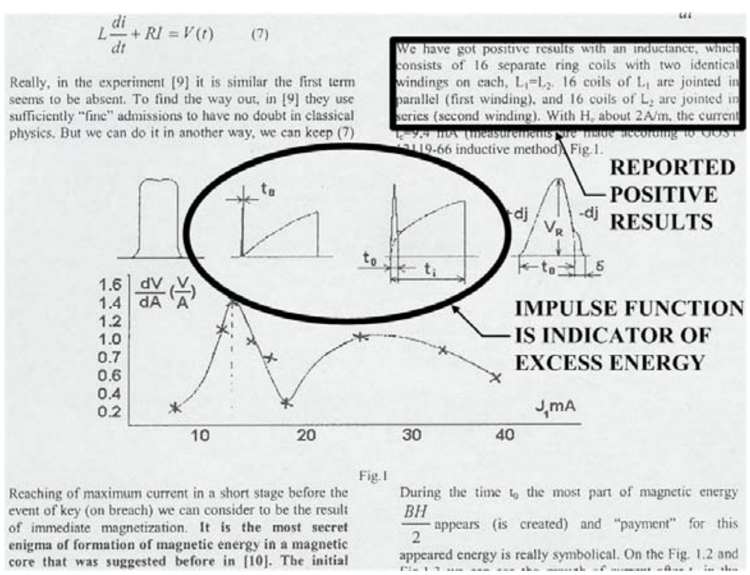


FIGURE 44. Excess free energy is harvested as NEGATIVE energy.

The diagram above shows that an impulse function occurs when excess residual magnetic flux is found in the core of a coil being magnetized at the start of the next cycle, $t = 0\,\text{sec}$. If harvested, the energy of this function manifests as NEGATIVE energy and couples to POSITIVE energy forming a *complex* direct current. This current consists of both *real* and *imaginary* components where the *real* current i is time-forward and the *imaginary* current $j\,i$ is time-future. The *real* current component is considered to be classic HOT CURRENT and the *imaginary* current component is considered to be COLD CURRENT.

Depending upon how much residual flux is available in the core, the energy in this impulse function could be quite substantial.



$$L\frac{di}{dt} + RI = V(t) \qquad (7)$$

Really, in the experiment [9] it is similar the first term seems to be absent. To find the way out, in [9] they use sufficiently "fine" admissions to have no doubt in classical physics. But we can do it in another way, we can keep (7)

We have got positive results with an inductance, which consists of 16 separate ring coils with two identical windings on each, $L_1=L_2$. 16 coils of L_1 are jointed in parallel (first winding), and 16 coils of L_2 are jointed in series (second winding). With H_c about 2A/m, the current ... 119-66 inductive method) Fig.1.

Fig.1

Reaching of maximum current in a short stage before the event of key (on breach) we can consider to be the result of immediate magnetization. **It is the most secret enigma of formation of magnetic energy in a magnetic core that was suggested before in [10]. The initial**

During the time t_0 the most part of magnetic energy $\frac{BH}{2}$ appears (is created) and "payment" for this appeared energy is really symbolical. On the Fig. 1.2 and

FIGURE 45. Excess energy found in N. Zaev's device.

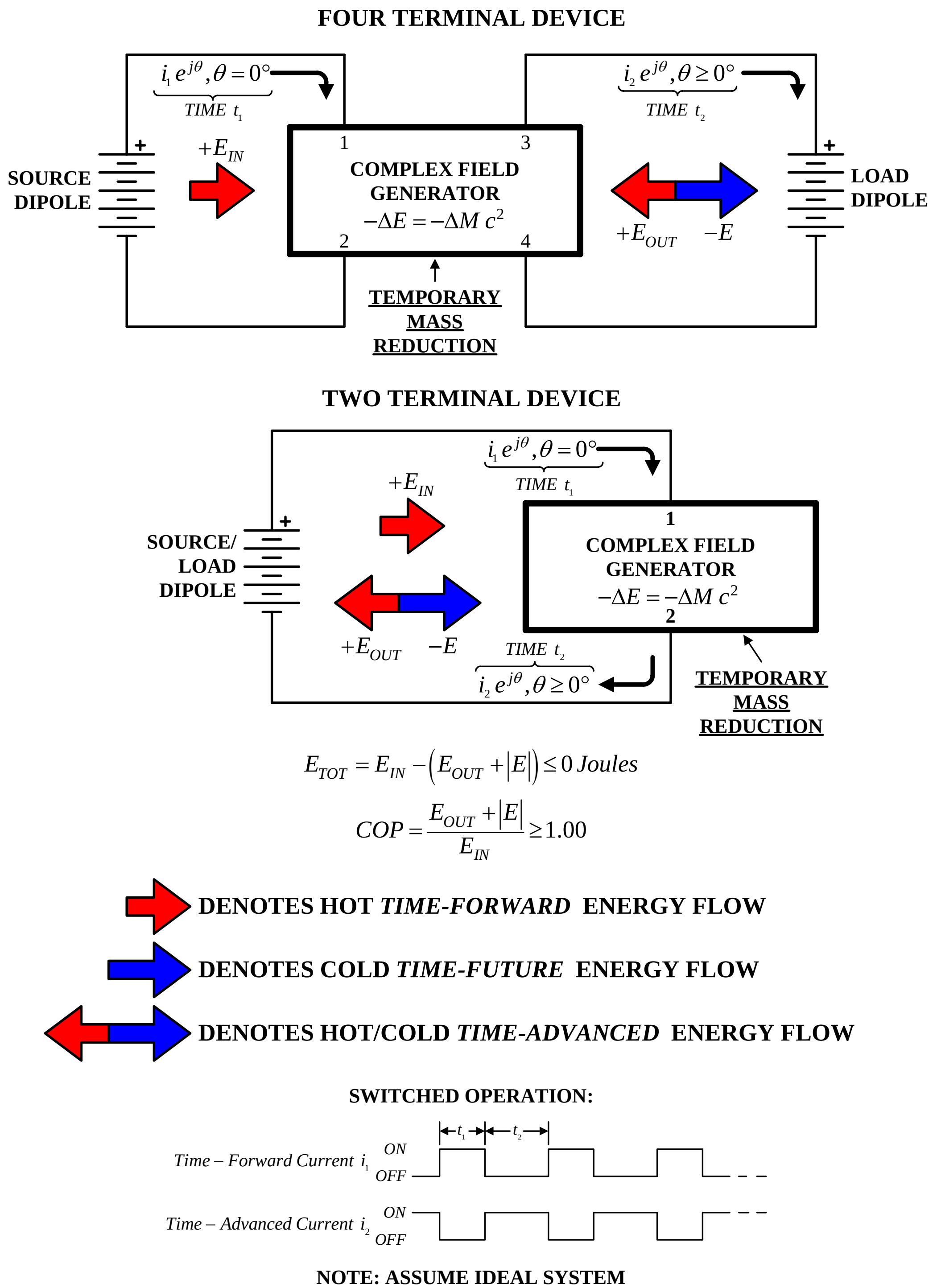


FIGURE 46. Two terminal / four terminal *complex* field mass fluctuating systems.

The diagram above shows a typical configuration of four terminal and two terminal *complex* field mass fluctuating systems. In the four terminal systems, energy from the source dipole (i.e., a battery) enters through terminals 1 and 2. Excess energy leaves through terminals 3 and 4 and charges the load dipole. In the two terminal systems, the source dipole also acts as the load dipole. Energy leaves the source dipole through terminals 1 and 2 and excess energy leaves through the same terminals at Time t_2 later.

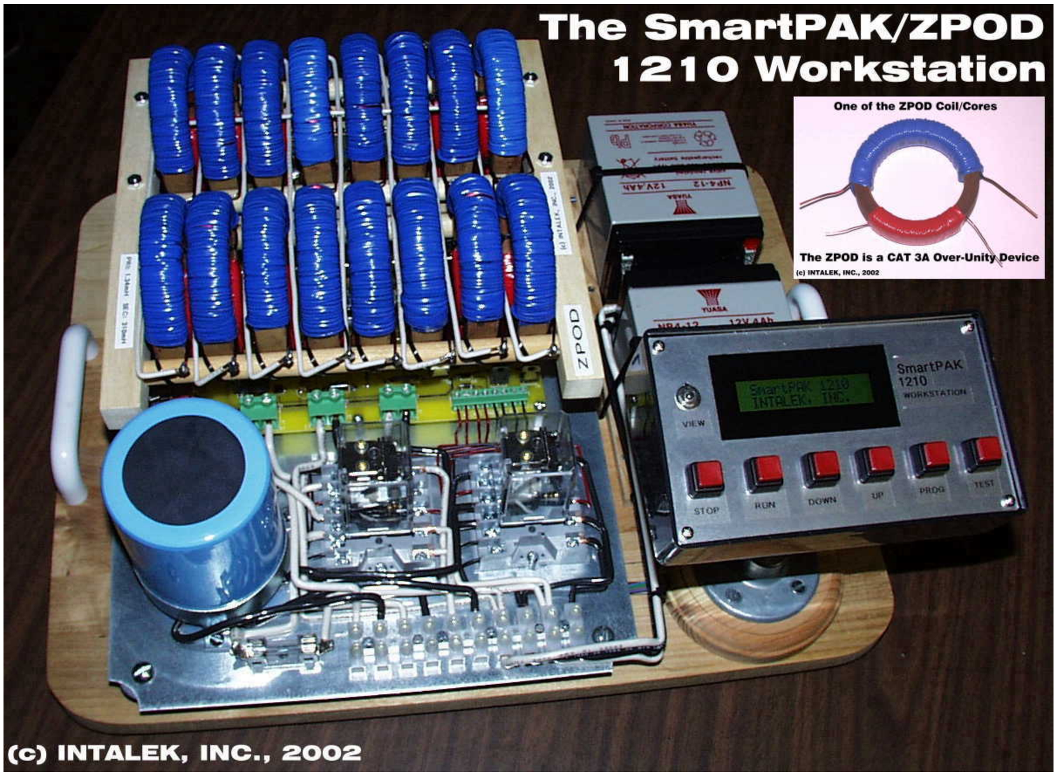


FIGURE 47. The SmartPAK/ZPOD 1210 Workstation.

FIGURE 48. The ZPOD in operation.

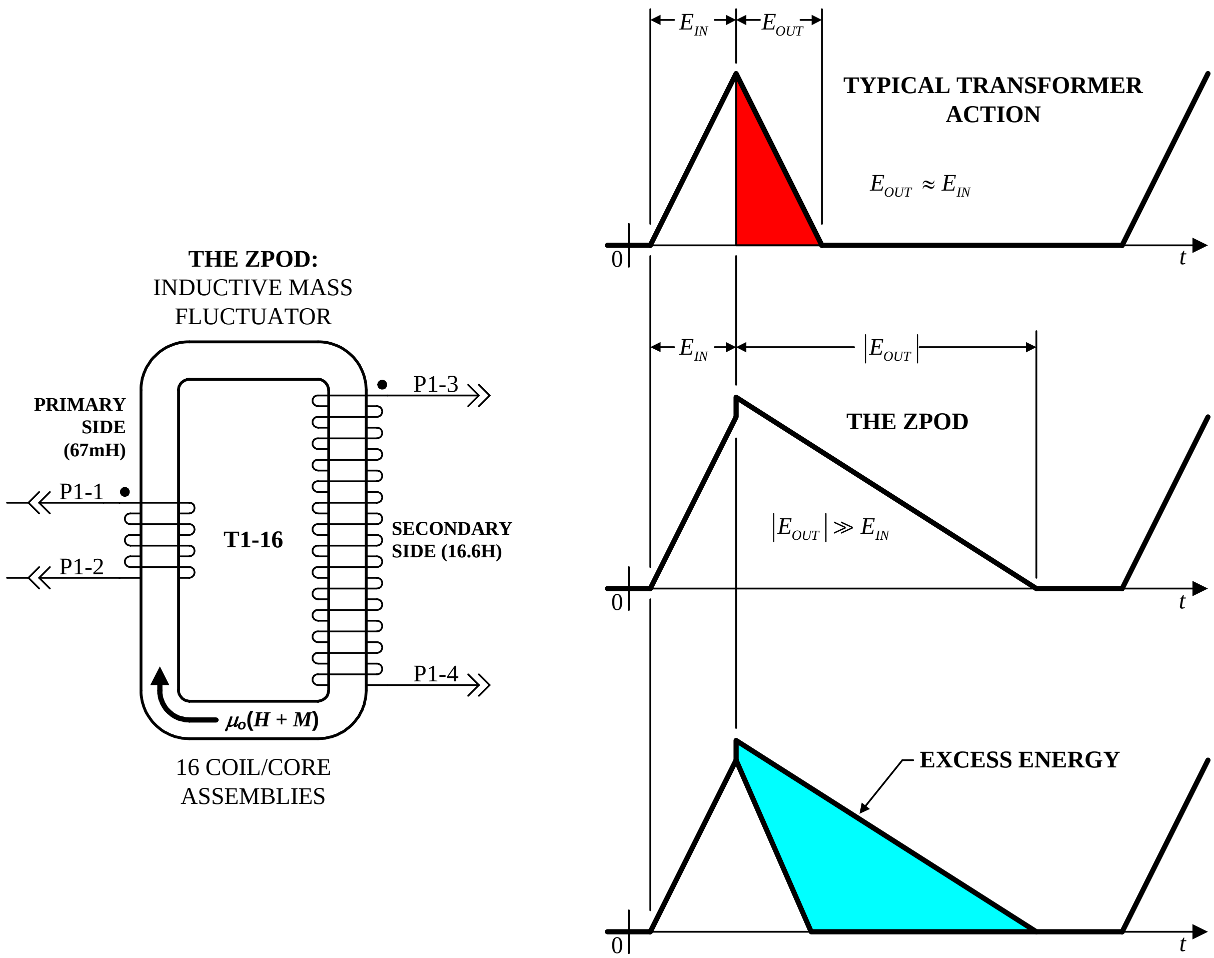


FIGURE 49. The excess energy of the ZPOD.

FIGURE 50. The eBike using SmartPAK technology.

FIGURE 51. The eBike using a SmartPAK 3610-30.

TESLA'S COMPLEX FIELD GENERATOR

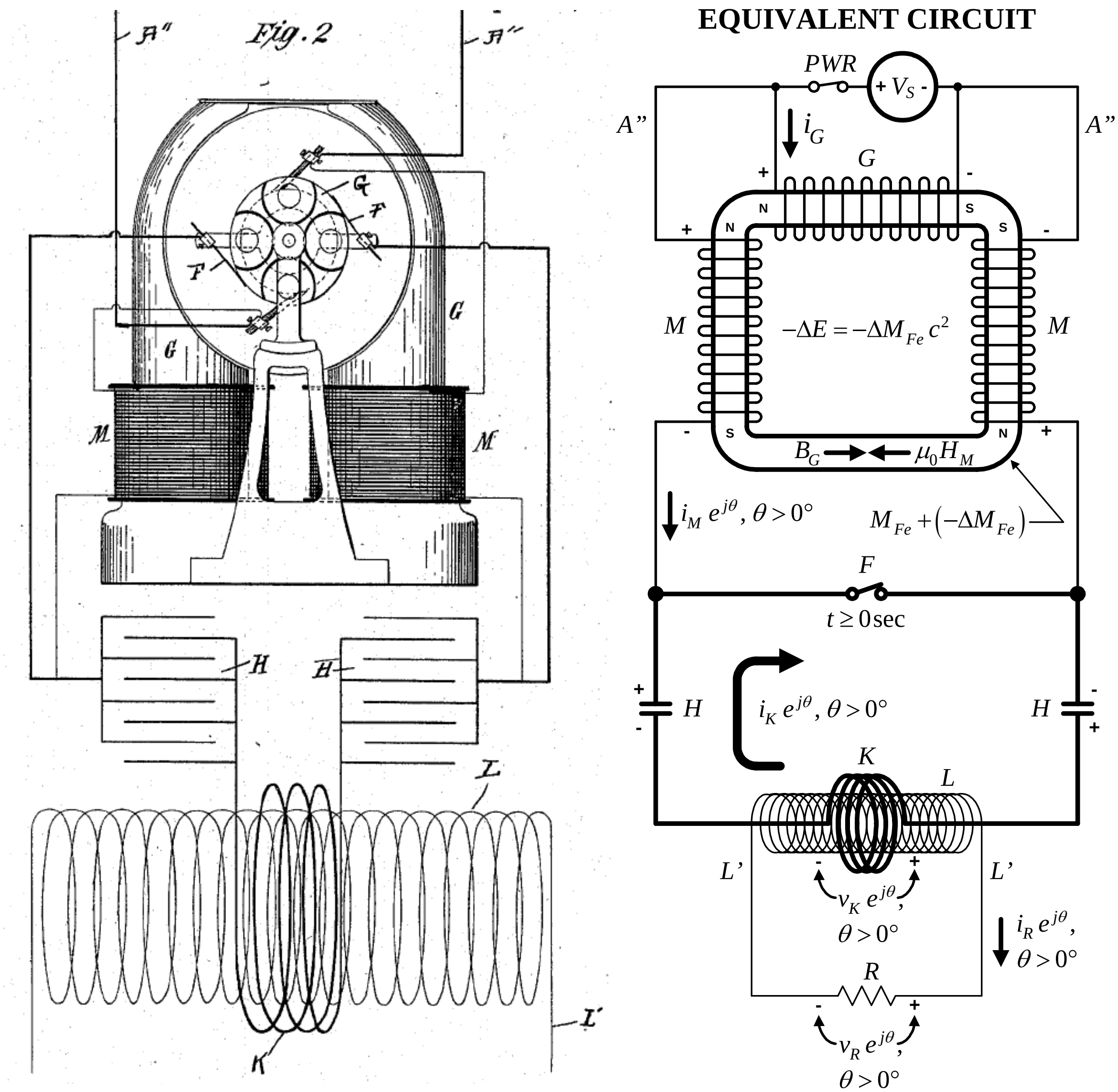


FIGURE 52. Nikola Tesla's US patent 568,176.

Nikola Tesla was the first to develop the phenomenon of complex fields back in the 1880's. He devised a series of machines patented in the 1890's that greatly amplify this phenomenon, which he later called RADIANT ENERGY. As shown above, the pivoting magnetic domains created by Amperian Currents of the ferromagnetic material are ordered in the direction of field B_G by magnetizing coil G. Magnetizing the high inductance coils M create an opposing field $\mu_0 H_M$ that acts upon the ordered domains of the material, thus canceling or partially canceling the *real* magnetic field created by the Amperian Currents. An *imaginary* magnetic field $j B_M$ emerges due to this cancellation and couples back into the magnetizing direct current as $i_M e^{j\theta}$, where $\theta > 0°$. Therefore, the magnetizing direct current becomes *complex* because the circulating motions of the electrons are rotating into the *imaginary* axis.

As shown above, before switch F is closed, the capacitors H are charged with a ***complex* direct current** $i_M e^{j\theta}$ produced by an opposing flux from coils M. The *complex* field energy is stored in capacitors H. At the moment of switch F closure $t = 0\sec$, the ***complex* direct current** flows through coil K, rapidly discharging capacitors H. A very large *complex* electric potential $v_L e^{j\theta}$ is observed across the secondary coil L.

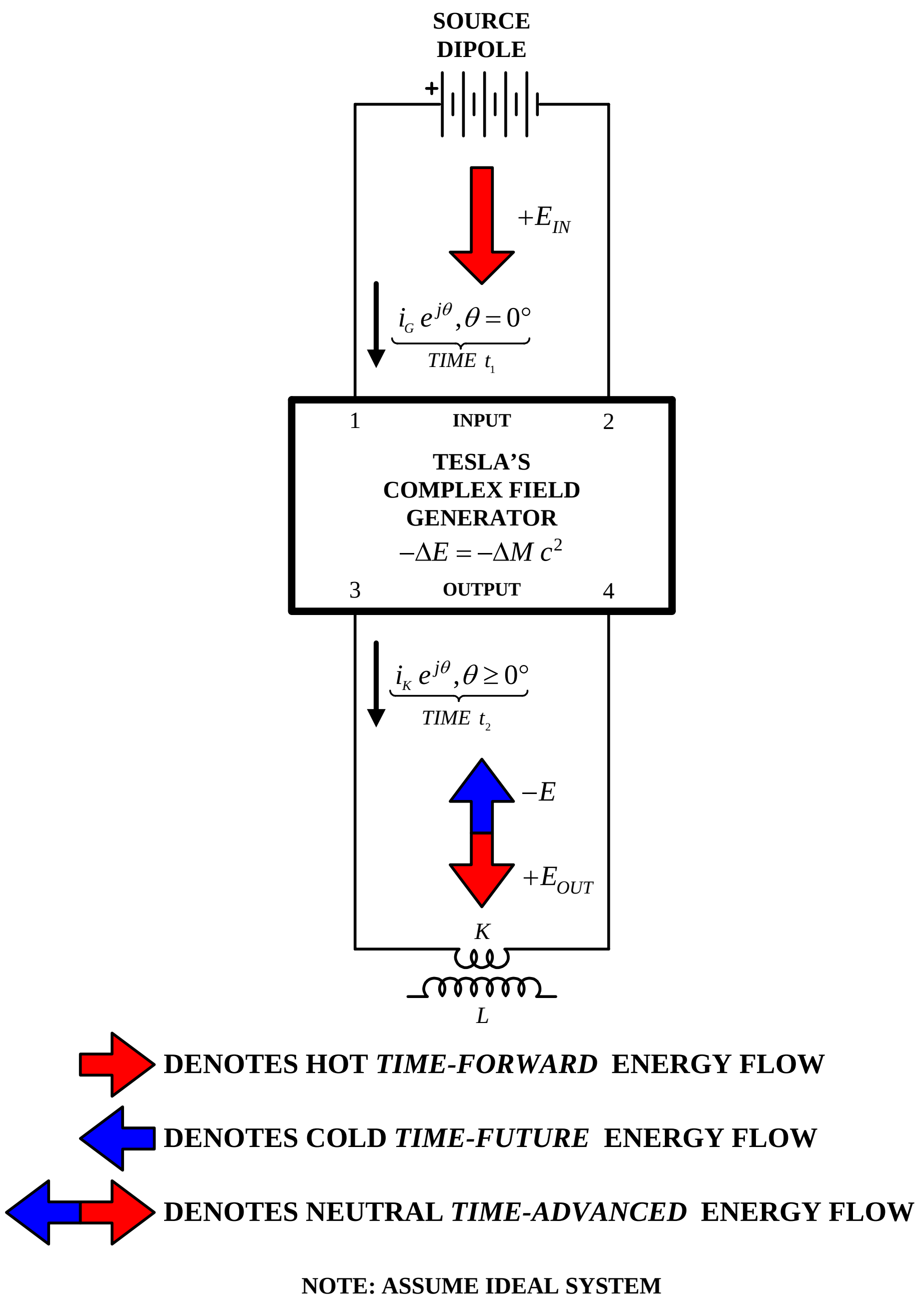


FIGURE 53. Tesla's four terminal *complex* field generator.

THE HUTCHISON EFFECT EXPLAINED

FIGURE 54. Metal samples from John Hutchison's lab.

As shown above, John Hutchison successfully applied the Tesla *complex* field to metal samples with amazing results. These bulk metal samples were melted at room temperature without any application of heat. The *complex* fields induced cold eddy currents within the metal, which in turn, caused the metal to cold melt. As the metal softened, John inserted bits of other metals and organic material as shown. With the field turned off, the metal re-solidified trapping these materials within the metal lattice structure.

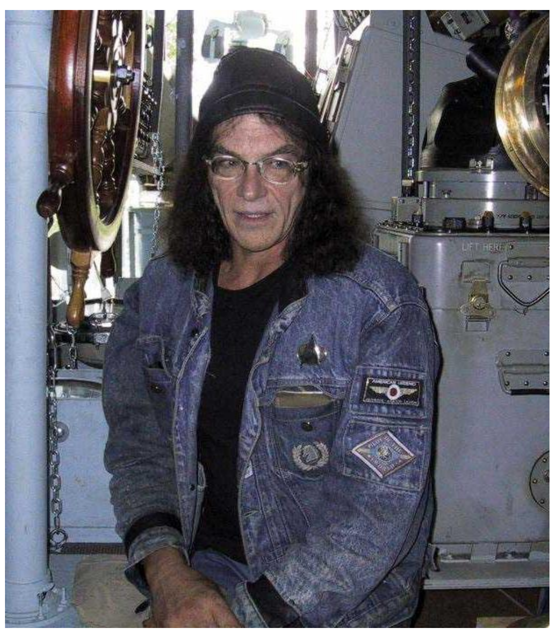


FIGURE 55. John Hutchison in his lab.

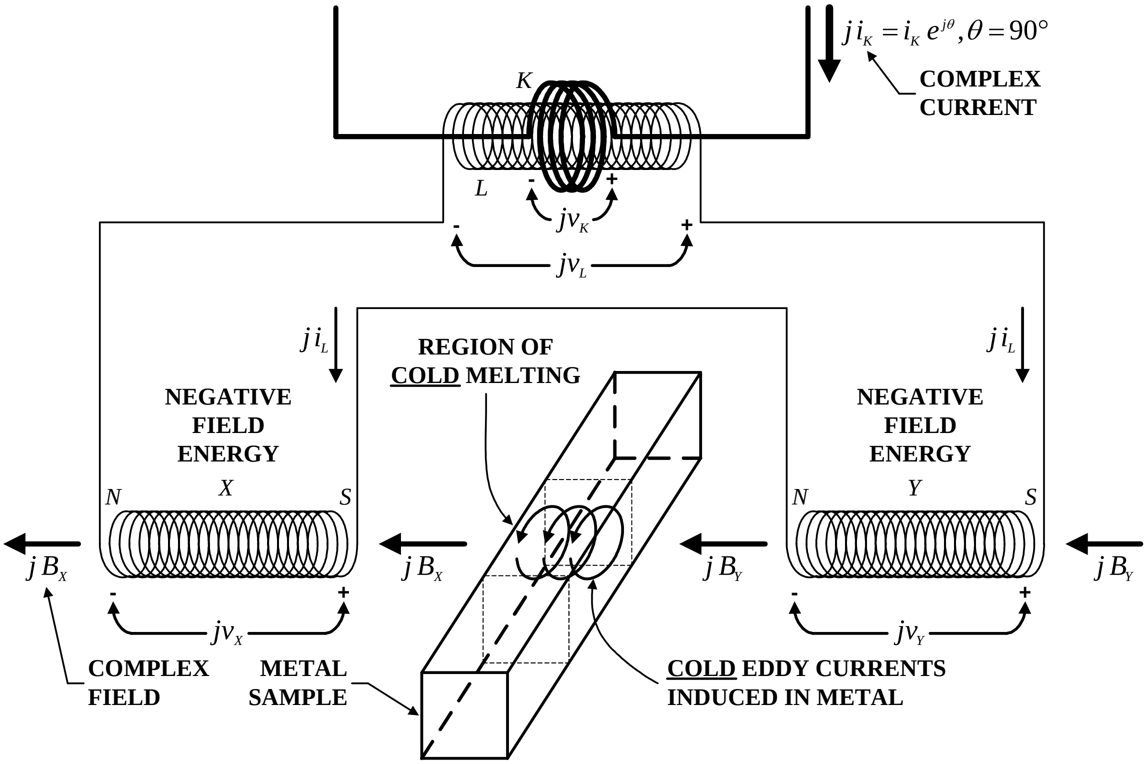


FIGURE 56. Cold eddy currents being induced in a metal sample.

As shown above, cold eddy currents are induced in the metal block with the application of *complex* magnetic fields. A *complex* current flowing through the coils produces these magnetic fields. The magnetic field energy surrounding these coils is NEGATIVE, and the metal sample in the presence of this field will cold melt due to induction.

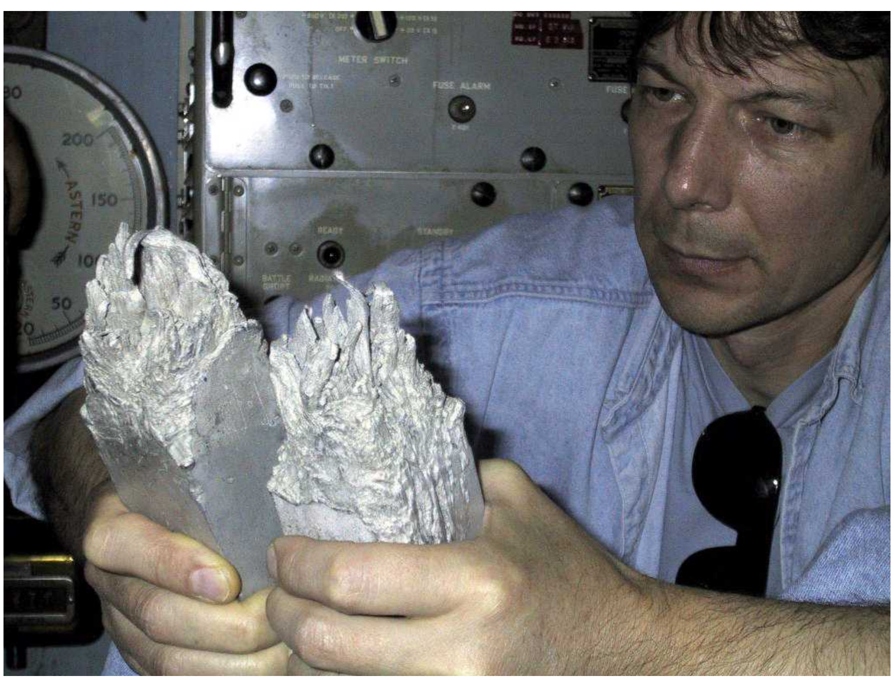


FIGURE 57. Another cold melted metal sample.

A CONCEPT VEHICLE THAT UTILIZES GMF

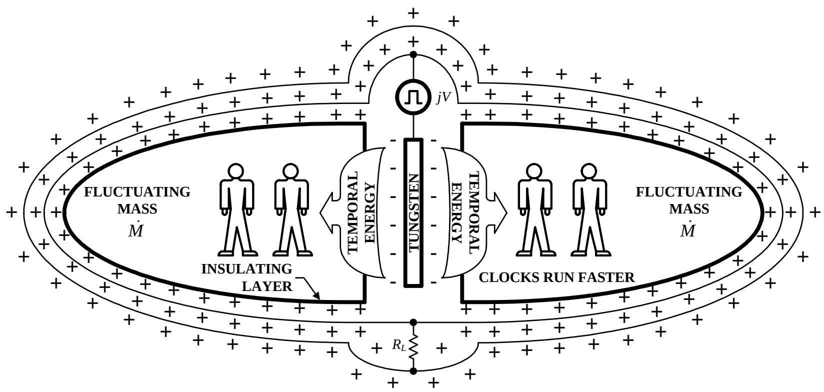


FIGURE 58. A cutaway view of a capacitive-based mass fluctuating concept vehicle.

Shown above is a concept vehicle for **tunneling through vast distances of space**. The first step concerning this antigravitational system is the disassociation of tungsten metal with the application of a *complex* voltage jV . This metal is used as a fuel source of gravitational energy bound as mass within the element. As the metal disassociates, it radiates away this gravitational energy in the form of a temporal field within the vehicle. This causes clocks to speedup and the mass of the vehicle to decrease. The second step begins by switching off the disassociating process, thereby turning off the temporal field. Mass converted to electrical energy is amplified as field energy in the outer hull, which can be stored, radiated away, or used to power the vehicle. The vehicle seeks a new equipotential surface of gravity that corresponds to its' new mass, thus producing lift. Then, the cycle begins again.

ENERGIZING THE GRAVITATIONAL PROPULSION UNIT (GPU)

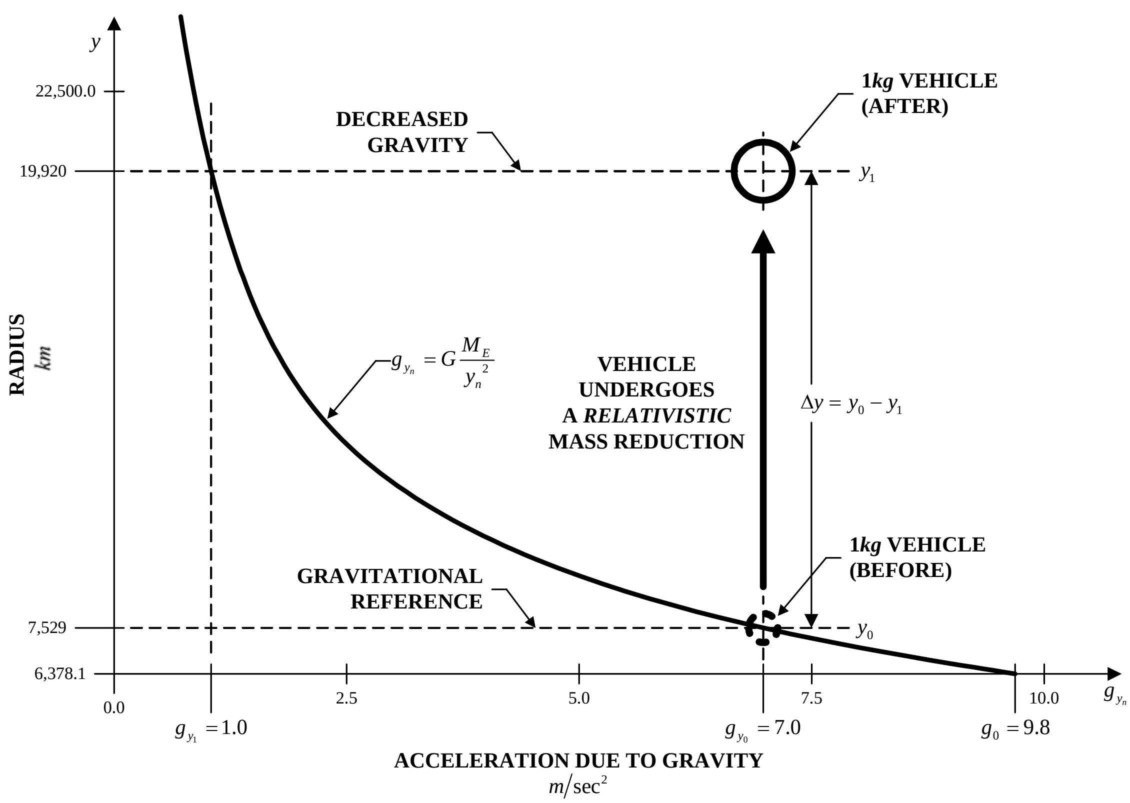


FIGURE 59. Vehicle is undergoing a negative gravitational mass fluctuation above the Earth.

The diagram above shows a $1kg$ vehicle undergoing a negative gravitational mass fluctuation in a given an equipotential surface of gravity reference. A typical gravity profile of the Earth shown above is based on Newton's gravity. This system decreases the *relativistic* mass of the vehicle such that it displaces or vectors in height to a new equipotential surface of gravity above the Earth.

Deactivating this system causes the same vehicle to *naturally* fall based on universal mass attraction.

Example 30. Assuming there are no other gravitational influences besides the Earth and given a vehicle of mass $1kg$ positioned at an initial radius y_0 as shown in the diagram above, compute the new gravitational mass M_{y_1} of the vehicle displaced Δy away from the Earth.

So, given,

- Mass of vehicle $M_{y_0} = 1.0\,kg$
- Vehicle at initial radius $y_0 = 7.529 \times 10^6\,m$
- Displacement of vehicle $\Delta y = y_0 - y_1 = -12.391 \times 10^6\,m$
- Speed of light $c = 2.99792458 \times 10^8\,m/\sec$
- Gravitational constant $G = 6.67260 \times 10^{-11}\,N\,m^2/kg^2$
- Mass of the Earth $M_E = 5.9787 \times 10^{24}\,kg$

The final radius y_1 of the vehicle above the Earth is,

$$y_1 = y_0 - \Delta y = \left(7.529 \times 10^6\,m\right) - \left(-12.391 \times 10^6\,m\right) = 19.920 \times 10^6\,m \tag{697}$$

The acceleration due to gravity at altitude $y_0 = 7.529 \times 10^6\,m$ above the Earth is,

$$g_{y_0} = \frac{G\,M_E}{y_0^{\,2}} = \frac{\left(6.67260 \times 10^{-11}\,N\,m^2/kg^2\right)\left(5.9787 \times 10^{24}\,kg\right)}{\left(7.529 \times 10^6\,m\right)^2} = 7.038\,m/\sec^2 \tag{698}$$

The acceleration due to gravity at altitude $y_1 = 19.920 \times 10^6\,m$ above the Earth is,

$$g_{y_1} = \frac{G\,M_E}{y_1^{\,2}} = \frac{\left(6.67260 \times 10^{-11}\,N\,m^2/kg^2\right)\left(5.9787 \times 10^{24}\,kg\right)}{\left(19.920 \times 10^6\,m\right)^2} = 1.005\,m/\sec^2 \tag{699}$$

Given the exponential solution of the *natural relativistic* mass model, the new gravitational mass is,

$$M_{y_1} = M_{y_0}\,e^{\left(\frac{g_{y_1}\,y_1 - g_{y_0}\,y_0}{c^2}\right)} = \left(1.0\,kg\right)e^{\left(\frac{\left(1.005\,m/\sec^2\right)\left(19.920\times10^6\,m\right) - \left(7.038\,m/\sec^2\right)\left(7.529\times10^6\,m\right)}{\left(2.99792458\times10^8\,m/\sec\right)^2}\right)} \tag{700}$$

$$M_{y_1} = 0.9999999996333\,kg \tag{701}$$

The gravitational mass was reduced by,

$$M_{y_1} - M_{y_0} = \left(0.9999999996333\,kg\right) - \left(1.0\,kg\right) = -3.667 \times 10^{-10}\,kg \tag{702}$$

CLOSED-LOOP GRAVITATIONAL FLIGHT CONTROL SYSTEM

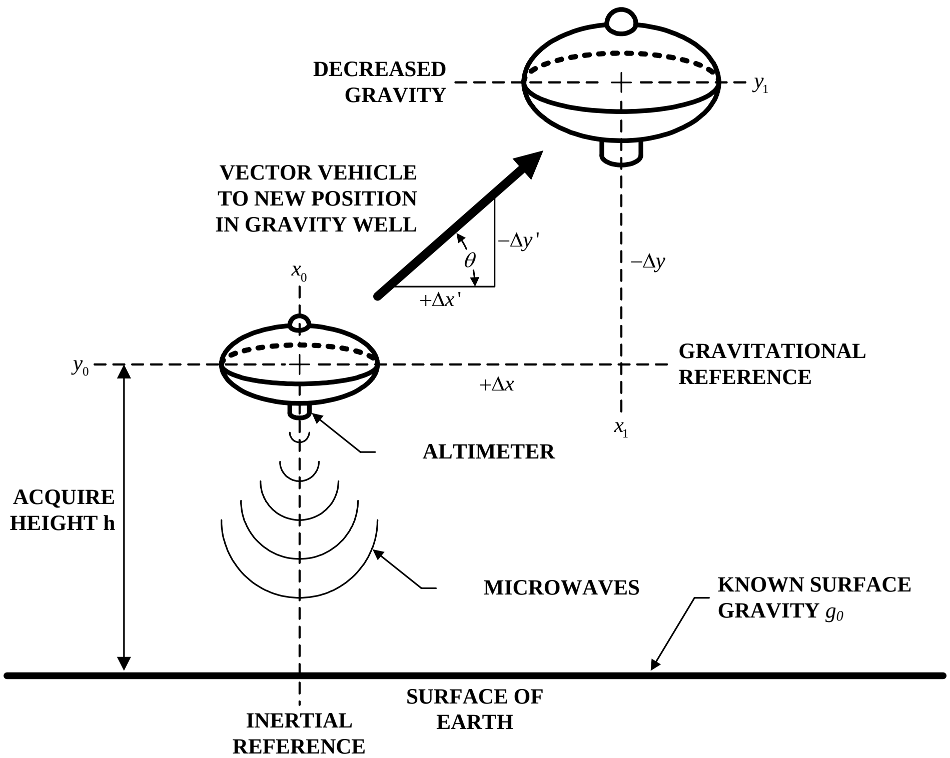


FIGURE 60. Acquire position information, then energize the GPU and vectored to new position.

The first step for a Gravitational Flight Control System, or GFCS, is to acquire vehicle height *h* information above a known surface gravity g_0. Once this height information is acquired, the second step involves calculating the current mass fluctuation $\dot{M}_{cur}$. The third step requires the pilot to determine what the next desired position will be, so, predicted mass fluctuation $\dot{M}_{pre}$ is calculated and transmitted to a Gravitational Propulsion Unit, or GPU. The GPU is energized causing the mass of the vehicle to fluctuate, and in turn, vectors to the desired position. This system can run "closed-loop" by implementing a software algorithm called a high-speed Proportional-Integral-Derivative (PID) control loop. It acquires new height information and computes the error difference between current and desired position. The PID calculates and transmits in real-time an error value to the GPU. The error value sent to the GPU determines the rate of mass fluctuation. This rate *may* exceed the speed of light because the vehicle isn't traversing space by an inertial means, but by a gravitational means. Therefore, warp factor equation shown by Whitfield (1968) could be used. The warp factor W equation is,

$$v = W^3 c \tag{703}$$

$$W = \sqrt[3]{\frac{|v|}{c}} = \sqrt[3]{\frac{\left|\frac{\Delta y}{T}\right|}{c}} \tag{704}$$

Where, W is the warp factor, c is the speed of light, and v is the velocity.

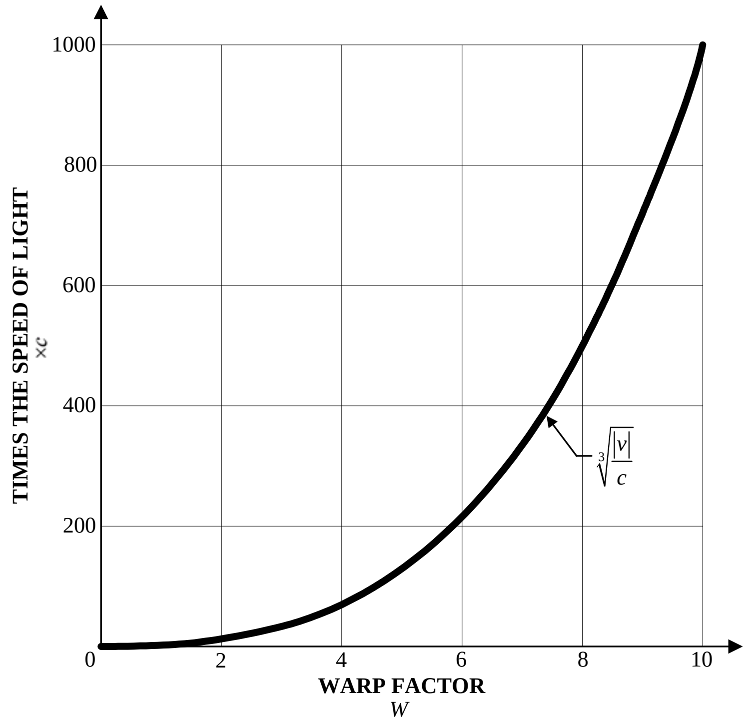


FIGURE 61. The warp factor vs. the speed of light.

Example 31. Assuming there are no other gravitational influences besides the Earth and given a GPU with a NEGATIVE mass fluctuation rate $\Delta M_{RATE}(t)$ operating for a period of time T, compute the displacement Δy of a vehicle leaving from the surface of the Earth at warp factor W with a vehicle mass of $1.0\,kg$.

So, given,

Initial mass of vehicle $M_{y_0} = 1.0\,kg$

Mass fluctuation rate $\Delta M_{RATE}(t) = -1.0\times10^{-4}\ gm/\sec$

Operating time $T = 1.0\times10^{-3}\,s\,\mathrm{ec}$

Vehicle located at initial radius $Y_0 = 6.3781\times10^{6}\,m$

Speed of light $c = 2.99792458\times10^{8}\ m/\sec$

Gravitational constant $G = 6.67260\times10^{-11}\ N\,m^2/kg^2$

Mass of the Earth $M_E = 5.9787\times10^{24}\ kg$

The new gravitational mass of the vehicle M_{y_1} operating a GPU for a period of time T is,

$$M_{y_1} = M_{y_0} + \left(\Delta M_{RATE}(t)\times T\right) = \left(1.0\,kg\right) + \left[\left(-1.0\times10^{-4}\ gm/\sec\right)\left(1.0\times10^{-3}\ \sec\right)\right] \tag{705}$$

$$M_{y_1} = 0.9999999999000\,kg \tag{706}$$

Given the exponential form of the *natural relativistic* mass $M_{\Delta y}$ model, compute the new radius y_1 within a given gravity well g_y is,

$$M_{y_1} = M_{y_0}\, e^{\left(\frac{g_{y_1}\, y_1 - g_{y_0}\, y_0}{c^2}\right)} = M_{y_0}\, e^{\left(\frac{\frac{G\,M_E}{y_1} - \frac{G\,M_E}{y_0}}{c^2}\right)} \tag{707}$$

$$\frac{M_{y_1}}{M_{y_0}} = e^{\left(\frac{\frac{G\,M_E}{y_1} - \frac{G\,M_E}{y_0}}{c^2}\right)} \tag{708}$$

$$c^2 \ln\left(\frac{M_{y_1}}{M_{y0}}\right) = \frac{G\,M_E}{y_1} - \frac{G\,M_E}{y_0} \tag{709}$$

$$y_1 = \frac{y_0}{1 + \frac{y_0\, c^2}{G\,M_E} \ln\left(\frac{M_{y_1}}{M_{y_0}}\right)} \tag{710}$$

$$y_1 = \frac{\left(6.3781\times10^6\, m\right)}{1 + \frac{\left(6.3781\times10^6\, m\right)\left(2.99792458\times10^8\, m/\text{sec}\right)^2}{\left(6.67260\times10^{-11}\, N\, m^2/kg^2\right)\left(5.9787\times10^{24}\, kg\right)} \ln\left(\frac{0.9999999999000\, kg}{1.0\, kg}\right)} \tag{711}$$

$$y_1 = 7.4484\times10^6\, m \tag{712}$$

$$\Delta y = y_0 - y_1 = \left(6.3781\times10^6\, m\right) - \left(7.4484\times10^6\, m\right) \tag{713}$$

$$\Delta y = -1.0703\times10^6\, m \tag{714}$$

The warp factor W is,

$$W = \sqrt[3]{\frac{|v|}{c}} = \sqrt[3]{\frac{\left|\frac{\Delta y}{T}\right|}{c}} = \sqrt[3]{\frac{\left|\frac{\left(-1.0703\times10^6\, m\right)}{\left(1.0\times10^{-3}\,\text{sec}\right)}\right|}{\left(2.99792458\times10^8\, m/\text{sec}\right)}} == \sqrt[3]{\frac{\left|-1.0703\times10^9\, m/\text{sec}\right|}{\left(2.99792458\times10^8\, m/\text{sec}\right)}} \tag{715}$$

$$W = 1.5284 \tag{716}$$

Leaving from the surface of the Earth, the vehicle is displaced over 665 miles above the Earth within 1 millisecond.

CONCLUSION

The parameters of space and time identified as mass, volume, frequency, time, temperature, and energy are functions of gravity. Therefore, controlling the mass of a space vehicle, for example, controls gravity, but more precisely, controls its' current position within a given gravity well. The control of how fast mass fluctuates controls the speed of the vehicle through this well. Whether the vehicle implementing GMF is above the surface of the Earth or traveling at warp factor 1.53 through deep space, the physical constants where shown in previous sections to remain invariant within the vehicle.

The vehicles' Gravitational Propulsion Unit, or GPU, using one of the two devices that are under investigation control mass by an electrical means, thus utilizing the theoretical link between gravity and electromagnetism presented in this paper. Either device can change its' own mass by converting it to excess electrical energy, and as a consequence, the broken symmetry of mass-energy conservation causes the gravitational energy equivalent of mass to be radiated away as a temporal field. A force is created and acts antigravitationally on the device. This changes the frequency of clocks within the vehicle *relative* to clocks outside of the vehicle.

The vehicle will require a Gravitational Flight Control System, or GFCS, capable of controlling the GPU. This control system acts as an interface between man and the GPU. The development of this system can be directly implemented from the mathematical formulations in this paper. An additional requirement is the development of real-time navigation software programs that maps the gravity of the entire Sol System, and includes the nearby stars.

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